Transformations, Domain, and Range of Quadratic Functions

Concepts
- Determining domain and range from a graph
- Describing the changes in the graph of a quadratic parent function when one or more parameters are changed
- Determining the equation of a quadratic function when transformations are described
- Completing the square to convert a quadratic function from standard form to vertex form

Technology Goals
Using the Transformation Graphing Application with quadratic functions

Overview
In this activity, participants will use the TI-84 Plus to explore transformations of quadratic functions.

Materials
- TI-84 Plus
- Transformation Graphing Application (Transfrm)

Part 1
1. Sketch the graph of each function by hand, and describe the transformation of the parent function \( f(x) = x^2 \).
2. Check with a graphing device.
   \[ f(x) = -(x - 3)^2 + 2 \]
   \[ f(x) = 3(x + 2)^2 - 1 \]
   \[ f(x) = 0.5(x + 1)^2 - 2 \]

Part 2
Use the graphs of these functions to determine the domain and range of each function.

Part 3: Applying Transformations
1. Given the function \( y = x^2 \), sketch the graph, and write an algebraic representation for the new function determined by the following sequence of transformations:
   - A horizontal shift left 4 units.
   - A vertical stretch with a factor of 2.
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- A vertical shift down 5 units.
2. Provide graphical support for the result.

**Part 4: Applying Transformations—Converting to Graphing Form**

\[ f(x) = 4x^2 + 16x + 17 \]

1. Rewrite the given function in “vertex form”
   \[ f(x) = a(x - h)^2 + k, \]
   and then sketch the graph.
   - Is this a good reason to teach completing the square?

**Part 5: Making Transformations Algebraically**

\[ f(x) = 4(x + 2)^2 + 1 \]

1. This graph is a transformation of \( y = x^2 \), which would include the points (0, 0), (1, 1), and (-1, 1).
2. These points will be transformed as follows:
   - A vertical stretch of 4 will multiply all \( y \)-values by 4.
   - Then a vertical slide of 1 will add 1 to all \( y \)-values.
   - A horizontal slide left 2 units will subtract 2 from each \( x \)-value.

<table>
<thead>
<tr>
<th>Orig. Pt.</th>
<th>Algebraic Transformations</th>
<th>New Pt.</th>
</tr>
</thead>
<tbody>
<tr>
<td>(0, 0)</td>
<td>(0 - 2, 0*4+1)</td>
<td>(-2,1)</td>
</tr>
<tr>
<td>(1, 1)</td>
<td>(1 - 2, 1*4 + 1)</td>
<td>(-1, 5)</td>
</tr>
<tr>
<td>(-1, 1)</td>
<td>(-1 - 2, 1*4+1)</td>
<td>(-3, 5)</td>
</tr>
</tbody>
</table>

**Part 6: Use the Transfrm Application**

Investigate the graph of \( y = Ax^2 + Bx + C \) when A and C are fixed and B varies.
1. Press [APPS], and select Transfrm.
2. Press [Y=], and enter the function as written above.
4. Fix the value of A at 1 and the value of C at 0.
5. Change the values of B by hand by pressing \[ \uparrow \downarrow \].
6. Set the PLAY (>) option to run values for B from –4 to 4, with steps of 1.
7. Change to FAST PLAY (>>).
8. Turn the TrailOn.

**Part 7: Optional Activity**

1. Use the *Transfrm* application to investigate
\[ y = A(x - C)^2 + D \]

Note: We can use only A, B, C, and D as variables. Unfortunately, that means that we cannot use h and k as our books do. C and D were selected here so that they would conform to the later trig functions, which can be written as
\[ y = A \text{fct}[B(x - C)] + D \, . \]

**Part 8: Guess My Coefficients APP**

1. Link the *Guess My Coefficients* APP to participants so they can play the game, or send it via the TI-Navigator™ system.
2. With the calculator turned on, press [APPS], select *GuesCoef*, and press [ENTER]. See Figure 1.
3. Press any key to move along the opening screens to the Select a Game screen.
   - Options, such as [INFO], appear at the bottom of several screens. See Figure 2.
   - Press the graphing keys under the appropriate selection to activate the option.
4. On the Select a Game screen, press [OPTIONS] to set up the number of problems (up to 99) or play until a user misses three problems.
   - To select actions, such as Grid options, use the arrow keys and press [\[] (ZOOM)] to change the options.
   - Press [BACK] to return to the Select a Game screen. See Figure 3.
5. Select 1:LINEAR on the Select a Game screen, then choose 1:\(y = mx + b\) or slope-intercept form. See Figure 4.

6. Press any key to view the graph as shown in Figure 5.
7. Study the graph, and then press any key.

8. Input the correct values as shown in Figure 6.

9. The graph will draw using the entered values.
   - If the values are correct, a message will display as shown in Figure 7.
   - To move on, press [OK] (GRAPH).

10. The next problem is displayed. Press any key to see the graph, and press again to get the input box as shown in Figure 8.
    - If the answer is incorrect, the box will reappear. A check mark will show the correct response.

11. Press [Hint] to see two points on the graphs and to trace the graph as shown in Figure 9.
12. Input the correct value for \( m \) as shown in Figure 10.

13. Press [OK] to continue play. See Figure 11.
14. Press \([\text{2nd}] \: [\text{QUIT}]\), and follow instructions to quit play.
15. Play to see who gets the high score.

16. Figures 12-14 show all function forms available for this game.

**To Quit**

1. Press \([\text{2nd}] \: [\text{QUIT}]\) at any time to access previous screens and to leave the application.

For a classroom-ready format of the Guess My Coefficients activity, including student handouts and worksheets, visit the Activities Exchange online using the following address: [http://education.ti.com/educationportal/activityexchange/activity_list.do?cid=us](http://education.ti.com/educationportal/activityexchange/activity_list.do?cid=us). Search for ‘Guess Coefficients.’