Bounce Back: The Exponential Pattern of Rebound Height

Concepts

- Maximum heights for a bouncing ball
- Modeling the bounce height data with an exponential function.

Materials

- TI-84 Plus
- CBR 2™ Motion Detector or Go!™ Motion and direct calculator cable
- Unit-to-CBR 2™ or I/O unit-to-unit cable
- EasyData™ application
- Ball (a basketball works well)

Overview

In this exercise, you will collect motion data for a bouncing ball using a Motion Detector. You will then analyze this data to test the model \( y = hp^x \).

Introduction

When a ball bounces up and down on a flat surface, the maximum height it reaches decreases from bounce to bounce. In fact, the maximum height decreases in a very predictable way for most types of balls.

The relationship between the maximum height attained by the ball on a given bounce will be called the rebound height. The number of bounces that have occurred since the ball was released is an exponential \( y = hp^x \) where \( y \) represents the rebound height, \( x \) represents the bounce number, \( h \) is the release height, and \( p \) is a constant that depends on the physical characteristics of the ball used.

It is easy to see where this model originated. Suppose that the ball is released from height \( h \). Then on each bounce it rebounds to a fraction \( p \) of the previous maximum height. After zero, one and two bounces, the ball will attain a maximum height of \( h, hp, (hp)p = hp^2 \), and so forth. The relation above is generalized for any \( x \) number of bounces.
Procedure

1. Set up the Motion Detector and calculator.
   - Open the pivoting head of the Motion Detector. If your Motion Detector has a sensitivity switch, set it to Normal as shown in Figure 1.
   - Turn the calculator on, and make sure it is on the Home Screen.
   - Connect it to the Motion Detector. This may require the use of a data-collection interface.

2. Position the Motion Detector about 1.5m above the floor so that the disc is pointing straight down. See Figure 2.

3. Set up EasyData™ for data collection.
   - Start the EasyData™ application if it is not already running.
   - Select File from the Main screen, and then select New to reset the application.
   - Select Setup from the Main screen, and then select Ball Bounce.

4. Practice dropping the ball so that it bounces straight up and down beneath the Motion Detector.
   - Minimize the ball’s sideways travel. Dropping the ball from about waist high works well.
   - The ball must never get closer than 0.5m to the detector.
   - Be sure to pull your hands away from the ball after you drop it so the Motion Detector does not detect your hands.

5. Select Start to begin data collection.
   - The Ball Bounce instructions will appear on the calculator screen. Follow the instructions to collect data.

6. When the data collection is complete, a graph of distance versus time will be displayed.

7. Examine the distance versus time graph.
   - The distance versus time graph should contain a series of at least five smoothly changing parabolic regions.
   - If you need to repeat the data collection, select Main and repeat Step 5.

8. Once you are satisfied with the distance versus time data, select Main.

9. Exit EasyData™ by selecting Quit, and then selecting OK.
Analysis

1. Display a graph of ball height above the floor versus time.
   - Press \[2\text{nd} [\text{STAT PLOT}].
   - Press \[\text{ENTER}\] to select Plot1, and press \[\text{ENTER}\] again to select On.
   - Press \[\text{ZOOM}\].
     Press 9:ZoomStat to display a graph with the \(x\) and \(y\) ranges set to fill the screen with data.

2. Press [TRACE] to determine the coordinates of a point on the graph using the cursor keys.

3. To compare the distance data to the model for bounce height, read the maximum height of each bounce from your distance versus time data.
   - Do this by tracing across the graph using the cursor keys.
   - Start with the initial release height, and call that bounce number zero.
   - Record the consecutive maximum rebound heights for the next five bounces.
   - Round these values to the nearest 0.001m.

4. Enter the information into lists on the calculator.
   - Press \[\text{STAT}\] and \[\text{ENTER}\] to see the data lists.
   - Press \[\text{A}\] to highlight the L1 header, and press \[\text{CLEAR}\] to clear the list.
   - Press \[\text{ENTER}\] to move to the first element of L1.
   - Enter the bounce numbers, starting from 0 and ending with 5. Press \[\text{ENTER}\] after each entry.
   - Use \[\text{\text{l}}\] and \[\text{\text{r}}\] to move to the header of L2. Press \[\text{CLEAR}\] to clear the list.
   - Press \[\text{ENTER}\] to move to the first element of L2.

5. Now that you have the data to be plotted stored in lists in the calculator, you are ready to plot the rebound height versus bounce number.
   - Press \[2\text{nd} [\text{STAT PLOT}].\] and press [ENTER] to select Plot1.
   - Change the Plot1 settings to match the screen shown in Figure 3.
   - Press [ENTER] to select any of the settings you change.
   - Press [ZOOM], and then select 9:ZoomStat to draw a graph with the \(x\) and \(y\) ranges set to fill the screen with data.
The graph you see is rebound height versus bounce number. The model is an exponential function: \( y = hp^x \), where \( y \) is the rebound height, and \( x \) the bounce number.

The \( h \) and \( p \) are two parameters that you need to determine. The \( h \) represents the starting height (sometimes called the zeroeth bounce), or the height before the first bounce. That is the height on the \( x = 0 \) line of the Data Table, which is also the \( y \)-intercept.

To determine the parameter \( p \), you can plot the model equation and try different values for \( p \) until one fits best.

1. First, you need to enter the exponential model.
   - Press \([Y=]\).
   - Press \( \text{CLEAR} \) to remove any existing equation.
   - Enter the expression \( H*P^X \) into the \( Y_1 \) field.
   - Press \( \text{2nd} \) [QUIT] to return to the Home Screen.
2. Set a value for the parameters \( h \) and \( p \), and then look at the resulting graph.
3. To obtain a good fit, you will need to try several values for \( p \).
   - Use the steps below to store different values to the parameters \( h \) and \( p \).
   - Start with \( p = 1 \). Experiment until you find one that provides a good fit for the data.
4. Enter the value for the parameter \( h \), which is the starting height.
   - Press \( \text{2nd} \) \( H \) [ENTER] to store the value into the variable \( H \).
5. Enter a value for the parameter \( p \).
   - Press \( \text{2nd} \) \( P \) [ENTER] to store the value into the variable \( P \).
6. Press \( \text{GRAPH} \) to see the data with the model graph superimposed.
7. Press \( \text{2nd} \) [QUIT] to return to the Home Screen.
   - You will not need to change the value for \( h \) further, but the best value for \( p \) will be something smaller than 1.
   - Try a variety of values for \( p \) until you get a good fit to the experimental data.
Creating a Linear Equation

The exponential equation \( y = hp^x \) can be made into a linear equation by taking the log of both sides:

\[
\begin{align*}
\ln y &= \ln(hp^x) \\
&= \ln h + \ln(p^x) \\
&= x \ln p + \ln h
\end{align*}
\]

That is, a graph of \( \ln y \) versus \( x \) is linear with a slope of \( \ln p \) and \( y \)-intercept of \( \ln h \). You can use the data collected so far to make this a linear graph.

1. To do this, you need to create a data list containing the natural log of the rebound heights.
   - Press \( \boxed{\text{LN}} \).
   - Press \( \boxed{2nd} \) \( \boxed{L2} \).
   - Press \( \boxed{1} \) to close the \( \ln \) function.
   - Press \( \boxed{\text{STO} \bullet} \), and press \( \boxed{2nd} \) \( \boxed{L2} \) to complete the expression \( \ln(L2) \rightarrow L2 \).
   - Press \( \boxed{\text{ENTER}} \) to perform the calculation.

2. Display a graph of \( \ln(\text{rebound height}) \) versus bounce number.
   - Press \( \boxed{\text{ZOOM}} \).
   - Press \( \boxed{9} \):ZoomStat to display a graph with the \( x \) and \( y \) ranges set to fill the screen with data.
   - Press \( \boxed{\text{TRACE}} \) to determine the coordinates of a point on the graph using the cursor keys.

3. Since the graph is nearly linear, have the calculator fit a line to the data.
   - Press \( \boxed{\text{STAT}} \) and move to the \( \text{CALC} \) menu.
   - Press \( \boxed{4} \):LinReg(ax+b) to copy the command to the Home Screen.
   - Press \( \boxed{2nd} \) \( \boxed{L1} \) \( \boxed{\bullet} \) \( \boxed{2nd} \) \( \boxed{L2} \) \( \boxed{\bullet} \) to enter the lists containing the data.
   - Press \( \boxed{\text{VARS}} \), and move to the Y-VARS menu.
   - Select 1:Function.
   - Press \( \boxed{\text{ENTER}} \) to copy \( Y1 \) to the expression.

On the home screen, you will now see the entry \( \text{LinReg}(ax+b) \ L1, L2, Y1 \). This command will perform a linear regression with \( L1 \) as the \( x \), \( L2 \) as the \( y \) values, and store the equation variable in \( Y1 \).

4. Press \( \boxed{\text{ENTER}} \) to perform the linear regression.

5. Press \( \boxed{\text{GRAPH}} \) to see the graph.