Common Core Investigations
Teacher’s Guide
Grade 8

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Connected Mathematics (CMP) is a field-tested and research-validated program that focuses on a few big ideas at each grade level. Students explore these ideas in depth, thereby developing deep understanding of key ideas that they carry from one grade to the next. The sequencing of topics within a grade and from grade to grade, the result of lengthy field-testing and validation, helps to ensure the development of students’ deep mathematical understanding and strong problem-solving skills. By the end of grade 8, CMP students will have studied all of the content and skills in the Common Core State Standards for Mathematics* (CCSSM) for middle grades (Grades 6–8). The focus on helping students develop deep mathematical understanding and strong problem solving skills aligns well to the intent of the Common Core State Standards for Mathematics, which articulates 3 to 5 areas of emphasis at each grade level from Kindergarten through Grade 8.

The sequence of content and skills in CMP varies in some instances from that in the CCSSM, so in collaboration with the CMP2 authors, Pearson has created a set of investigations for each grade level to further support and fully develop students’ understanding of the content standards of the CCSSM. The authors are confident that the CMP2 curriculum supplemented with the additional investigations at each grade level will address all of the content and skills of the CCSSM, but even more, will contribute significantly to advancing students’ mathematical proficiency as described in the Standards for Mathematical Practices of the CCSSM. Through the in-depth exploration of concepts, students become confident in solving a variety of problems with flexibility, skill, and insightfulness, and are able to communicate their reasoning and understanding in a variety of ways.

In this supplement, you will find support for all of the Common Core (CC) Investigations.

- The At-A-Glance page includes Teaching Notes and answers to all problems and exercises for the CC Investigation.
- The Additional Practice and Skill Practice pages can be reproduced for your students. These offer opportunities for students to reinforce the core concepts of the CC Investigation.
- Use the Check-Up to assess your students’ understanding of the concepts presented in the investigation.
- The answers for all of the ancillary pages are found at the back of this book.
- The reduced student pages are provided for your convenience as you read through the teaching support and plan for implementing each investigation.

In the Pacing Guide (pp. xii–xiii), we propose placement for teaching each CC Investigations. CC Investigations 1 and 2 build on the concepts of exponents and patterns explored in Growing, Growing, Growing. The transformations students master in Kaleidoscopes, Hubcaps, and Mirrors are further explored in CC Investigation 3, and CC Investigation 4 includes further geometry topics for study. After completing Samples and Populations, use CC Investigation 5 to explore bivariate data.
The following alignment of the Common Core State Standards for Mathematics (June 2, 2010 release) to Pearson’s *Connected Mathematics 2* (CMP2) ©2009 program includes the supplemental investigations that complete the CMP2 program.

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<thead>
<tr>
<th>COMMON CORE STATE STANDARDS GRADE 8</th>
<th>CMP2 UNITS</th>
<th>CONTENT</th>
</tr>
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<tbody>
<tr>
<td><strong>The Number System</strong></td>
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<tr>
<td>Know that there are numbers that are not rational, and approximate them by rational numbers.</td>
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</tr>
<tr>
<td>8.NS.1</td>
<td>Looking For Pythagoras</td>
<td>Inv. 4: Using the Pythagorean Theorem</td>
</tr>
<tr>
<td>Understand informally that every number has a decimal expansion; the rational numbers are those with decimal expansions that terminate in 0s or eventually repeat. Know that other numbers are called irrational.</td>
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</tr>
<tr>
<td>8.NS.2</td>
<td>Looking For Pythagoras</td>
<td>Inv. 4: Using the Pythagorean Theorem</td>
</tr>
<tr>
<td>Use rational approximations of irrational numbers to compare the size of irrational numbers, locate them approximately on a number line diagram, and estimate the value of expressions (e.g., ( \pi^2 )).</td>
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### COMMON CORE STATE STANDARDS GRADE 8

<table>
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<tr>
<th>Expressions and Equations</th>
<th>CMP2 UNITS</th>
<th>CONTENT</th>
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<tbody>
<tr>
<td><strong>8.EE.1</strong> Know and apply the properties of integer exponents to generate equivalent numerical expressions.</td>
<td>Growing, Growing, Growing</td>
<td>Inv. 5: Patterns With Exponents</td>
</tr>
<tr>
<td><strong>8.EE.2</strong> Use square root and cube root symbols to represent solutions to equations of the form $x^2 = p$ and $x^3 = p$, where $p$ is a positive rational number. Evaluate square roots of small perfect squares and cube roots of small perfect cubes. Know that $\sqrt{2}$ is irrational.</td>
<td>Looking For Pythagoras</td>
<td>Inv. 2: Squaring Off Inv. 3: The Pythagorean Theorem Inv. 4: Using the Pythagorean Theorem</td>
</tr>
<tr>
<td><strong>8.EE.3</strong> Use numbers expressed in the form of a single digit times an integer power of 10 to estimate very large or very small quantities, and to express how many times as much one is than the other.</td>
<td>Growing, Growing, Growing</td>
<td>Inv. 1: ACE 39–40 Inv. 2: ACE 15–17 Inv. 4: ACE 8 Inv. 5: ACE 56–60</td>
</tr>
<tr>
<td><strong>8.EE.4</strong> Perform operations with numbers expressed in scientific notation, including problems where both decimal and scientific notation are used. Use scientific notation and choose units of appropriate size for measurements of very large or very small quantities (e.g., use millimeters per year for seafloor spreading). Interpret scientific notation that has been generated by technology.</td>
<td>Growing, Growing, Growing</td>
<td>Inv. 5: ACE 56–57, 60</td>
</tr>
<tr>
<td><strong>8.EE.5</strong> Graph proportional relationships, interpreting the unit rate as the slope of the graph. Compare two different proportional relationships represented in different ways.</td>
<td>Thinking With Mathematical Models CC Investigations</td>
<td>Inv. 2: Linear Models and Equations CC Inv. 2: Functions</td>
</tr>
<tr>
<td><strong>8.EE.6</strong> Use similar triangles to explain why the slope $m$ is the same between any two distinct points on a non-vertical line in the coordinate plane; derive the equation $y = mx$ for a line through the origin and the equation $y = mx + b$ for a line intercepting the vertical axis at $b$.</td>
<td>Thinking With Mathematical Models CC Investigations</td>
<td>Inv. 2: Linear Models and Equations CC Inv. 2: Functions</td>
</tr>
<tr>
<td><strong>8.EE.7</strong> Solve linear equations in one variable.</td>
<td>Thinking With Mathematical Models Say It With Symbols</td>
<td>Inv. 2: Linear Models and Equations Inv. 1: Equivalent Expressions Inv. 2: Combining Expressions Inv. 3: Solving Equations</td>
</tr>
<tr>
<td><strong>8.EE.7.a</strong> Give examples of linear equations in one variable with one solution, infinitely many solutions, or no solutions. Show which of these possibilities is the case by successively transforming the given equation into simpler forms, until an equivalent equation of the form $x = a$, $a = a$, or $a = b$ results (where $a$ and $b$ are different numbers).</td>
<td>CC Investigations</td>
<td>Inv. 2: Linear Models and Equations CC Inv. 2: Functions</td>
</tr>
<tr>
<td><strong>8.EE.7.b</strong> Solve linear equations with rational number coefficients, including equations whose solutions require expanding expressions using the distributive property and collecting like terms.</td>
<td>Thinking With Mathematical Models Say It With Symbols</td>
<td>Inv. 2: Linear Models and Equations Inv. 1: Equivalent Expressions Inv. 2: Combining Expressions Inv. 3: Solving Equations Inv. 4: Looking Back at Functions</td>
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<tr>
<td>COMMON CORE STATE STANDARDS GRADE 8</td>
<td>CMP2 UNITS</td>
<td>CONTENT</td>
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<tr>
<td>8.EE.8 Analyze and solve pairs of simultaneous linear equations.</td>
<td>The Shapes of Algebra</td>
<td>Inv. 2: Linear Equations and Inequalities Inv. 3: Equations With Two or More Variables Inv. 4: Solving Systems of Linear Equations Symbolically</td>
</tr>
<tr>
<td>8.EE.8.a Understand that solutions to a system of two linear equations in two variables correspond to points of intersection of their graphs, because points of intersection satisfy both equations simultaneously.</td>
<td>The Shapes of Algebra</td>
<td>Inv. 2: Linear Equations and Inequalities Inv. 3: Equations With Two or More Variables Inv. 4: Solving Systems of Linear Equations Symbolically</td>
</tr>
<tr>
<td>8.EE.8.b Solve systems of two linear equations in two variables algebraically, and estimate solutions by graphing the equations. Solve simple cases by inspection.</td>
<td>The Shapes of Algebra</td>
<td>Inv. 1: ACE 56–57 Inv. 2: Linear Equations and Inequalities Inv. 3: Equations With Two or More Variables Inv. 4: Solving Systems of Linear Equations Symbolically</td>
</tr>
<tr>
<td>8.EE.8.c Solve real-world and mathematical problems leading to two linear equations in two variables.</td>
<td>The Shapes of Algebra</td>
<td>Inv. 2: Linear Equations and Inequalities Inv. 3: Equations With Two or More Variables Inv. 4: Solving Systems of Linear Equations Symbolically</td>
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<tr>
<td>Common Core State Standards Grade 8</td>
<td>CMP2 Units</td>
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<tr>
<td><strong>Functions</strong></td>
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<tr>
<td><strong>Define, evaluate, and compare functions.</strong></td>
<td><strong>8.F.1</strong> Understand that a function is a rule that assigns to each input exactly one output. The graph of a function is the set of ordered pairs consisting of an input and the corresponding output. NOTE Function notation is not required in Grade 8.</td>
<td><strong>CC Investigations</strong></td>
</tr>
<tr>
<td><strong>8.F.2</strong> Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions).</td>
<td><strong>Thinking With Mathematical Models</strong></td>
<td><strong>Inv. 1: Exploring Data Patterns</strong></td>
</tr>
<tr>
<td><strong>8.F.3</strong> Interpret the equation ( y = mx + b ) as defining a linear function, whose graph is a straight line; give examples of functions that are not linear.</td>
<td><strong>Thinking With Mathematical Models</strong></td>
<td><strong>Inv. 2: Linear Models and Equations</strong></td>
</tr>
<tr>
<td><strong>8.F.4</strong> Construct a function to model a linear relationship between two quantities. Determine the rate of change and initial value of the function from a description of a relationship or from two ((x, y)) values, including reading these from a table or from a graph. Interpret the rate of change and initial value of a linear function in terms of the situation it models, and in terms of its graph or a table of values.</td>
<td><strong>Thinking With Mathematical Models</strong></td>
<td><strong>Inv. 1: Exploring Data Patterns</strong></td>
</tr>
<tr>
<td><strong>8.F.5</strong> Describe qualitatively the functional relationship between two quantities by analyzing a graph e.g., where the function is increasing or decreasing, linear or nonlinear). Sketch a graph that exhibits the qualitative features of a function that has been described verbally.</td>
<td><strong>Thinking With Mathematical Models</strong></td>
<td><strong>Inv. 2: Linear Models and Equations</strong></td>
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**Use functions to model relationships between quantities.**
**COMMON CORE STATE STANDARDS GRADE 8**

**CMP2 UNITS**

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<th>CONTENT</th>
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<tbody>
<tr>
<td>Geometry</td>
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</table>

Understand congruence and similarity using physical models, transparencies, or geometry software.

**8.G.1**

Verify experimentally the properties of rotations, reflections, and translations:

| Kaleidoscopes, Hubcaps, and Mirrors |
| CC Investigations |
| Inv. 1: Three Types of Symmetry |
| Inv. 2: Symmetry Transformations |
| Inv. 3: Exploring Congruence |
| Inv. 4: Applying Congruence and Symmetry |
| Inv. 5: Transforming Coordinates |
| CC Inv. 3: Transformations |

**8.G.1.a**

Lines are taken to lines, and line segments to line segments of the same length.

| Kaleidoscopes, Hubcaps, and Mirrors |
| CC Investigations |
| Inv. 1: Three Types of Symmetry |
| Inv. 2: Symmetry Transformations |
| Inv. 3: Exploring Congruence |
| Inv. 4: Applying Congruence and Symmetry |
| Inv. 5: Transforming Coordinates |
| CC Inv. 3: Transformations |

**8.G.1.b**

Angles are taken to angles of the same measure.

| Kaleidoscopes, Hubcaps, and Mirrors |
| CC Investigations |
| Inv. 1: Three Types of Symmetry |
| Inv. 2: Symmetry Transformations |
| Inv. 3: Exploring Congruence |
| Inv. 4: Applying Congruence and Symmetry |
| Inv. 5: Transforming Coordinates |
| CC Inv. 3: Transformations |

**8.G.1.c**

Parallel lines are taken to parallel lines.

| Kaleidoscopes, Hubcaps, and Mirrors |
| CC Investigations |
| Inv. 1: Three Types of Symmetry |
| Inv. 2: Symmetry Transformations |
| Inv. 3: Exploring Congruence |
| Inv. 4: Applying Congruence and Symmetry |
| Inv. 5: Transforming Coordinates |
| CC Inv. 3: Transformations |

**8.G.2**

Understand that a two-dimensional figure is congruent to another if the second can be obtained from the first by a sequence of rotations, reflections, and translations; given two congruent figures, describe a sequence that exhibits the congruence between them.

| Kaleidoscopes, Hubcaps, and Mirrors |
| CC Investigations |
| Inv. 3: Exploring Congruence |

**8.G.3**

Describe the effect of dilations, translations, rotations, and reflections on two-dimensional figures using coordinates.

| Kaleidoscopes, Hubcaps, and Mirrors |
| CC Investigations |
| Inv. 2: ACE 24–25, 32 |
| Inv. 5: Transforming Coordinates |
| CC Inv. 3: Transformations |

**8.G.4**

Understand that a two-dimensional figure is similar to another if the second can be obtained from the first by a sequence of rotations, reflections, translations, and dilations; given two similar two dimensional figures, describe a sequence that exhibits the similarity between them.

| CC Investigations |
| CC Inv. 4: Geometry Topics |

**8.G.5**

Use informal arguments to establish facts about the angle sum and exterior angle of triangles, about the angles created when parallel lines are cut by a transversal, and the angle-angle criterion for similarity of triangles.

<p>| CC Investigations |
| CC Inv. 4: Geometry Topics |</p>
<table>
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<tr>
<td>Understand and apply the Pythagorean Theorem.</td>
<td>Looking For Pythagoras</td>
<td>Inv. 3: The Pythagorean Theorem</td>
</tr>
<tr>
<td><strong>8.G.6</strong> Explain a proof of the Pythagorean Theorem and its converse.</td>
<td>Looking For Pythagoras</td>
<td>Inv. 3: The Pythagorean Theorem</td>
</tr>
<tr>
<td><strong>8.G.7</strong> Apply the Pythagorean Theorem to determine unknown side lengths in right triangles in real-world and mathematical problems in two and three dimensions.</td>
<td>Looking For Pythagoras</td>
<td>Inv. 4: Using the Pythagorean Theorem</td>
</tr>
<tr>
<td><strong>8.G.8</strong> Apply the Pythagorean Theorem to find the distance between two points in a coordinate system.</td>
<td>Looking For Pythagoras</td>
<td>Inv. 2: Squaring Off \n Inv. 3: The Pythagorean Theorem</td>
</tr>
<tr>
<td>Solve real-world and mathematical problems involving volume of cylinders, cones, and spheres.</td>
<td>Kaleidoscopes, Hubcaps, and Mirrors \n Looking For Pythagoras \n Say It With Symbols \n CC Investigations</td>
<td>Inv. 1: ACE 47–49 \n Inv. 2: ACE 28 \n Inv. 3: ACE 24 \n Inv. 3: ACE 18–22, 25–26 \n Inv. 4: ACE 57–58 \n Inv. 1: ACE 55 \n Inv. 3: ACE 41 \n Inv. 4: ACE 39 \n CC Inv. 4: Geometry Topics</td>
</tr>
<tr>
<td>COMMON CORE STATE STANDARDS GRADE 8</td>
<td>CMP2 UNITS</td>
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<tr>
<td><strong>Statistics and Probability</strong></td>
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<tr>
<td>Investigate patterns of association in bivariate data.</td>
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<tr>
<td><strong>8.SP.1</strong></td>
<td>Construct and interpret scatter plots for bivariate measurement data to investigate patterns of association between two quantities. Describe patterns such as clustering, outliers, positive or negative association, linear association, and nonlinear association.</td>
<td><strong>Samples and Populations</strong></td>
</tr>
</tbody>
</table>
| **8.SP.2** | Know that straight lines are widely used to model relationships between two quantitative variables. For scatter plots that suggest a linear association, informally fit a straight line, and informally assess the model fit by judging the closeness of the data points to the line. | **Samples and Populations**  
**Thinking With Mathematical Models** | **Inv. 4: Relating Two Variables**  
**Inv. 2: Linear Models and Equations** |
| **8.SP.3** | Use the equation of a linear model to solve problems in the context of bivariate measurement data, interpreting the slope and intercept. | **The Shapes of Algebra**  
**Thinking With Mathematical Models** | **Inv. 2: Linear Equations and Inequalities**  
**Inv. 3: Equations With Two or More Variables**  
**Inv. 2: Linear Models and Equations**  
**Inv. 3: Inverse Variation** |
| **8.SP.4** | Understand that patterns of association can also be seen in bivariate categorical data by displaying frequencies and relative frequencies in a two-way table. Construct and interpret a two-way table summarizing data on two categorical variables collected from the same subjects. Use relative frequencies calculated for rows or columns to describe possible association between the two variables. | **CC Investigations** | **CC Inv. 5: Bivariate Data** |
This Pacing Guide offers suggestions as you look to implement the Grade 8 Common Core State Standards for Mathematics in the CMP2 classroom. The Chart shows placement recommendations for the Common Core Investigations provided in this supplement.

Investigations labeled as Review (R) offer timely practice of concepts from earlier grades, helping to activate students’ prior knowledge as they are introduced to new concepts that build on these concepts. Investigations labeled as Extending (*) offer students the opportunity to explore concepts in greater depth or to extend their study of concepts.

The suggested number of standard days for each unit is based on a 45-minute class period; a block period is assumed to be 90 minutes of instructional time. Common Core students entering Grade 8 were introduced to coordinate grids and data sets in Grade 6, and population sampling in Grade 7. Because of these prior understandings, students will begin the units Looking for Pythagoras and Samples and Populations with the tools needed to complete these units at an accelerated pace. This will leave time in the year to cover the CC Investigations needed for Grade 8.

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<tr>
<th>Thinking with Mathematical Models</th>
<th>Standard 18 days • Block 9 days</th>
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<td>Inv. 1 Exploring Data Patterns</td>
<td>8.F.2, 8.F.4, 8.F.5, 8.SP.3</td>
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<tr>
<td>Inv. 2 Linear Models and Equations</td>
<td>8.EE.6, 8.EE.7, 8.EE.7.b, 8.F.3, 8.F.5, 8.SP.2, 8.SP.3</td>
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<tr>
<td>Inv. 3 Inverse Variation</td>
<td>8.F.3</td>
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<tr>
<th>Looking for Pythagoras</th>
<th>Standard 18 ½ days • Block 9 ½</th>
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<tbody>
<tr>
<td>Inv. 1 Coordinate Grids</td>
<td>Reviews 6.NS.6</td>
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<tr>
<td>Inv. 2 Squaring Off</td>
<td>8.EE.2</td>
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<td>Inv. 3 The Pythagorean Theorem</td>
<td>8.EE.2, 8.G.6, 8.G.7, 8.G.8, 8.G.9</td>
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<tr>
<td>Inv. 4 Using the Pythagorean Theorem</td>
<td>8.NS.1, 8.NS.2, 8.EE.2, 8.G.7, 8.G.9</td>
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<tr>
<td>Inv. 1 Exponential Growth</td>
<td>8.EE.3, 8.F.2, 8.F.5</td>
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<td>Inv. 2 Examining Growth Patterns</td>
<td>8.EE.3, 8.F.5</td>
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<tr>
<td>Inv. 3 Growth Factors and Growth Rates</td>
<td>8.F.5</td>
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<tr>
<td>Inv. 4 Exponential Decay</td>
<td>8.EE.3</td>
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<tr>
<td>Inv. 5 Patterns with Exponents</td>
<td>8.EE.1, 8.EE.3, 8.EE.4, 8.F.3</td>
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<td>CC Inv. 1 Negative Exponents</td>
<td>8.EE.1, 8.EE.2</td>
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<tr>
<td>CC Inv. 2 Functions</td>
<td>8.EE.5, 8.EE.6, 8.EE.7.a, 8.F.1, 8.F.2</td>
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<tr>
<td><strong>Frogs, Fleas, and Painted Cubes</strong></td>
<td><strong>Standard 22 ½ days • Block 11 ½</strong></td>
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<tr>
<td>Inv. 1 Introduction to Quadratic Relationships</td>
<td>8.F.5</td>
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<td>Inv. 2 Quadratic Expressions</td>
<td>8.F.2, 8.F.5</td>
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<td>Inv. 3 Quadratic Patterns of Change</td>
<td>8.F.2, 8.F.5</td>
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<td>Inv. 4 What Is a Quadratic Function?</td>
<td>8.F.2, 8.F.5</td>
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<th><strong>Kaleidoscopes, Hubcaps, and Mirrors</strong></th>
<th><strong>Standard 32 days • Block 16</strong></th>
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<tr>
<td>Inv. 1 Three Types of Symmetry</td>
<td>8.G.1, 8.G.1.a, 8.G.1.b, 8.G.1.c, 8.G.9</td>
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<td>Inv. 2 Symmetry Transformations</td>
<td>8.G.1, 8.G.1.a, 8.G.1.b, 8.G.1.c, 8.G.3, 8.G.9</td>
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<td>Inv. 3 Exploring Congruence</td>
<td>8.G.1, 8.G.1.a, 8.G.1.b, 8.G.1.c, 8.G.2, 8.G.9</td>
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<td>Inv. 4 Applying Congruence and Symmetry</td>
<td>8.G.1, 8.G.1.a, 8.G.1.b, 8.G.1.c</td>
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<td>Inv. 5 Transforming Coordinates</td>
<td>8.G.1, 8.G.1.a, 8.G.1.b, 8.G.1.c, 8.G.3</td>
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<tr>
<td>CC Inv. 3 Transformations</td>
<td>8.GG.1, 8.GG.2, 8.GG.3</td>
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<td>CC Inv. 4 Geometry Topics</td>
<td>8.GG.3, 8.GG.4, 8.GG.5, 8.GG.9</td>
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<th><strong>Standard 21 days • Block 10 ½</strong></th>
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<td>8.EE.7, 8.EE.7.a, 8.EE.7.b, 8.G.9</td>
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<td>Inv. 2 Combining Expressions</td>
<td>8.EE.7, 8.EE.7.a, 8.EE.7.b, 8.F.2</td>
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<td>8.EE.7, 7.b, 8.G.9</td>
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<td>Inv. 4 Looking Back at Functions</td>
<td>8.EE.7.b, 8.G.9, 8.F.3, 8.F.4, 8.F.5</td>
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<td>Inv. 5 Reasoning With Symbols</td>
<td>Prepares for A-SSE-3</td>
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</tbody>
</table>

**KEY**

- ✓ Core Content
- R Review
- * Extending
CC Investigation 1: Exponents

Mathematical Goals

- Know and apply the properties of integer exponents to generate equivalent expressions.
- Use cube root symbols to represent solutions to equations of the form \(x^3 = p\).
- Evaluate cube roots of small perfect cubes.

Teaching Notes

The process for finding powers of rational numbers is the same as the process for finding powers of whole numbers. In this investigation, students will practice writing expressions involving repeated multiplication using exponents. Students will simplify both decimals and fractions raised to powers.

Students also will explore the rules of exponents to generate equivalent expressions involving both multiplication and division of exponents.

Negative exponents are difficult for many students to understand. Some students may simply write a negative sign in front of their solution. It is necessary to explain to the students why negative exponents represent repeated division. Discuss the fact that positive exponents indicate repeated multiplication. Extend that reasoning to show that negative exponents indicate repeated division. For example, since \(2^3 = 2 \times 2 \times 2\) and \(2^2 = 2 \times 2\), decreasing the exponent by 1 is equivalent to dividing by the base, 2. At this point, it may also be helpful to review the fact that any base raised to the zero power is equal to 1. Therefore, negative exponents involve repeated division.

The cubes and cube roots covered in this investigation are limited to those that students should be able to recognize or discover readily using repeated multiplication. They do not need to know the mechanics of finding the cube root of a greater number and will not need calculators.

Problem 1.1

Before Problem 1.1, review the Getting Ready and the rules for simplifying expressions involving multiplication or division of exponents. Ask:

- What do you do to multiply numbers with the same base? (Add the exponents.)
- What do you do to raise a power to a power? (Multiply the exponents.)
- What do you do to divide numbers with the same base? (Subtract the exponents.)

During Problem 1.1 B, ask: What are multiplicative inverses? (numbers that when multiplied together give a product of 1)
After Problem 1.1, ask: How do you write an equivalent expression for a number that has a negative exponent? (Take the reciprocal of the number and raise it to the positive exponent.)

Problem 1.2

Before Problem 1.2, point out the example of \( \sqrt[3]{8} = 2 \) and \( 2^3 = 8 \), and ask: How does this example show that finding a cube and finding a cube root are opposites? (When 2 is cubed, the result is 8; the cube root of 8 is 2.)

During Problem 1.2 B ask:

• How does the formula \( V = e^3 \) relate to the general formula for the volume of a rectangular prism, \( V = lwh \)? (For a cube, the length, width, and height all are the same, \( e \), so \( V = e \times e \times e = e^3 \).)

• How could making a table of the cubes of some numbers help you solve Part B2? (Show the cubes of numbers in a table until you find the maximum volume, 729.)

Summarize

To summarize the lesson, ask:

• How is finding the value of an exponent expression involving a decimal or fraction like finding the value of a similar expression with an integer base? (Use repeated multiplication for each.)

• When can you add the exponents to multiply numbers that have different exponents? (when the bases are the same)

• How can you use a multiplicative inverse to find the value of an expression with a negative exponent? (Find the value of the multiplicative inverse of the base raised to the positive exponent.)

• How do you find the cube root of a number? (Find a number that when multiplied by itself three times gives a product that is the original number.)
Assignment Guide for Investigation 1
Problem 1.1, Exercises 1–39
Problem 1.2, Exercises 40–52

Answers to Investigation 1

Getting Ready for Problem 1.1
- \(7^5 \times 7 = 7^{5+1} = 7^6\)
- \((12^5)^{11} = 12^{5 \times 11} = 12^{55}\)
- \(8^7 \div 8^3 = 8^{7-3} = 8^4\)

Problem 1.1
A. 1. \(2^4 = 2 \times 2 \times 2 \times 2 = 16; 2^3 = 2 \times 2 \times 2 = 8; 2^2 = 2 \times 2 = 4\)
2. The value is decreased by a factor of 2.
3. \(2^7 = 2^8 \div 2 = 256 \div 2 = 128\)
B. 1. \(2^4 = 2; 2^0 = 1; 2^{-1} = \frac{1}{2}; 2^{-2} = \frac{1}{4}; 2^{-3} = \frac{1}{8}; 2^{-4} = \frac{1}{16}\)
2. \(2^{-2} = \left(\frac{1}{2}\right)^2; 2^{-3} = \left(\frac{1}{2}\right)^3; 2^{-4} = \left(\frac{1}{2}\right)^4\)
3. a. \(4^{-2} = \left(\frac{1}{4}\right)^2 = \frac{1}{16}\)
b. \(\left(\frac{3}{5}\right)^{-3} = \left(\frac{5}{3}\right)^3 = \frac{125}{8} = 15\frac{5}{8}\)

Problem 1.2
A. \(\sqrt[3]{27} = 3; \sqrt[3]{125} = 5\)
B. 1. Find the cube root of both sides of the formula: \(V = e^3; e = \sqrt[3]{V}\).
2. | Volume (in.\(^3\)) | Edge Length (in.) |
<table>
<thead>
<tr>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>8</td>
<td>2</td>
</tr>
<tr>
<td>64</td>
<td>4</td>
</tr>
<tr>
<td>343</td>
<td>7</td>
</tr>
<tr>
<td>729</td>
<td>9</td>
</tr>
</tbody>
</table>
3. \(384 \div \frac{3}{4} = 512; \sqrt[3]{512} = 8\) in.

Exercises
1. \((3.6)^4\)
2. \(\left(\frac{4}{5}\right)^3\)
3. \((0.8)^5\)

4. \(\frac{1}{243}\)
5. 12.96
6. 0.064
7. about 113.4
8. \(\frac{4}{1125}\)
9. 0.0036
10. a. 0.1
    b. 0.01
    c. 0.001
    d. 0.0001
    e. The exponent increases by 1 while the value of the expression decreases by a factor of 10. Each term has a value that is 0.1 the value of the previous term.
    f. 0.00000001
11. \(7^{15}\)
12. \(3^{42}\)
13. \(235^{178}\)
14. \(17^{80}\)
15. \(5^{28}\)
16. \(2^{40}\)
17. \(x^{12}\)
18. \(m^7\)
19. \(p^{24}\)
20. \(\frac{1}{2}\)
21. \(\frac{1}{243}\)
22. \(\frac{1}{125}\)
23. \(\frac{5}{2}\)
24. \(\frac{27}{512}\)
25. \(\frac{100}{49}\)
26. \(\frac{1}{8}\)
27. 100
28. 196
29. 4
30. \(\frac{5}{81}\)
31. 96
32. 200
33. 0.02
34. 0.007
35. 7,000
36. 250,000
37. 0.000025

38. Yes; multiplying by the expression \(3^{-2}\) can be represented by multiplying by \(\frac{1}{9}\), or dividing by \(3^2\).

39. a. \(5^4 = 625; 5^3 = 125; 5^2 = 25; 5^1 = 5; 5^0 = 1\)
   b. \(10^4 = 10,000; 10^3 = 1,000; 10^2 = 100; 10^1 = 10; 10^0 = 1\)
   c. The value of any positive number with exponent zero is 1.

40. 1
41. 4
42. 6
43. 5
44. 10
45. 7
46. \(x = 5\)
47. \(x = 3\)
48. \(x = 6\)
49. \(x = 10\)
50. \(x = 1\)
51. \(x = 4\)
52. 7 in.
1. Scientists are working to develop a new type of glass that allows only \(\frac{1}{4}\) of the sun’s harmful rays to pass through. Two layers of this glass will allow \(\left(\frac{1}{4}\right)^2\) of the harmful rays to pass.
   a. What amount of harmful rays are able to pass through 4 layers of the glass? Show your work.

   b. In a biology lab, they need to protect the specimens from harmful rays. The director wants no more than \(\frac{1}{4,000}\) of the rays to reach his specimens. Will 6 layers of the new glass be enough? Explain.

   c. Another company is working on glass that allows in only \(\frac{1}{5}\) of the harmful rays per layer. Would using this glass allow the biology lab to use only 4 layers of the glass? Explain.

2. The table shows the volumes of some cube-shaped shipping cartons. Margaret needs to find room for the cartons in the warehouse.

<table>
<thead>
<tr>
<th>Carton</th>
<th>Volume (ft³)</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>27</td>
</tr>
<tr>
<td>B</td>
<td>64</td>
</tr>
<tr>
<td>C</td>
<td>125</td>
</tr>
<tr>
<td>D</td>
<td>216</td>
</tr>
</tbody>
</table>

   a. How many of carton C could Margaret fit in one row in a space that is 23 ft wide?

   b. How many of carton D could Margaret fit in one row in a space that is 40 ft wide?

   c. Carton B can be stacked no more than 3 cartons high. Margaret wants to fit 16 of the cartons in a space that is 23 ft wide. Will she be able to do that?

   d. Carton A can be stacked no more than 4 cartons high. How wide a space would Margaret need to keep 20 of those cartons?
Skill: Exponents with Whole Numbers and Decimals

Find the value of the expression.

1. \(8^3\)
2. \(2^5\)
3. \(5^4\)
4. \((10)^3\)
5. \((0.4)^3\)
6. \((2.5)^4\)
7. \((1.8)^4\)
8. \((6.2)^3\)
9. \(3^5\)
10. \(7^4\)
11. \((4.9)^3\)
12. \((0.5)^4\)

Skill: Exponents with Fractions

Find the value of the expression.

13. \(\left(\frac{1}{4}\right)^2\)
14. \(\left(\frac{1}{2}\right)^3\)
15. \(\left(\frac{2}{3}\right)^4\)
16. \(\left(\frac{5}{8}\right)^3\)
17. \(\left(\frac{1}{6}\right)^4\)
18. \(\left(\frac{4}{7}\right)^4\)
19. \(\left(\frac{1}{3}\right)^{-3}\)
20. \(\left(\frac{2}{5}\right)^3\)
21. \(\left(\frac{4}{3}\right)^{-2}\)
22. \(\left(\frac{1}{7}\right)^{-3}\)
23. \(\left(\frac{1}{8}\right)^4\)
24. \(\left(\frac{3}{2}\right)^{-4}\)
1. Find equivalent expressions to complete the table. The first row is done for you.

<table>
<thead>
<tr>
<th>Negative Exponent</th>
<th>Positive Exponent</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>(3^{-2})</td>
<td>((\frac{1}{3})^2)</td>
<td>(\frac{1}{9})</td>
</tr>
<tr>
<td>(4^{-5})</td>
<td>((\frac{1}{4})^4)</td>
<td></td>
</tr>
<tr>
<td>(7^{-2})</td>
<td></td>
<td></td>
</tr>
<tr>
<td>(8^{-2})</td>
<td>(4^2)</td>
<td></td>
</tr>
<tr>
<td>((\frac{1}{3})^{-3})</td>
<td></td>
<td></td>
</tr>
<tr>
<td>((\frac{1}{2})^{-5})</td>
<td>(5^4)</td>
<td></td>
</tr>
</tbody>
</table>

2. A charity notifies members by text message when there is a new event scheduled. The organizer sends a message to 4 members. When a member receives a message, they pass the message on to 4 other members. That pattern continues until everyone is notified.
   a. Write a multiplication expression and an equivalent expression using an exponent to show how many messages are sent out in the third round.

   b. How many rounds of texting will it take to alert all 600 members of a new event? Show how you found your answer.
3. Hiro uses the following containers in science class. The volume of each container is shown.

<table>
<thead>
<tr>
<th>Container</th>
<th>Volume</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>$18\pi \text{ in.}^3$</td>
</tr>
<tr>
<td>B</td>
<td>512 in.³</td>
</tr>
<tr>
<td>C</td>
<td>$144\pi \text{ in.}^3$</td>
</tr>
<tr>
<td>D</td>
<td>64 in.³</td>
</tr>
</tbody>
</table>

a. Containers B and D are cubes. Find the lengths of each container’s sides.

b. Containers A and C are half-spheres. The formula for the volume of a half-sphere is $V = \frac{2}{3}\pi r^3$. Find the radius of each container.

c. Hiro fills container C with water and uses about $\frac{3}{4}$ of the water to fill another cube-shape container. To the nearest inch, how long is each side of the cube?

d. Hiro has another half-sphere container. He fills and empties container B almost 3 full times to fill the half-sphere. To the nearest inch, what is the radius of the half-sphere?
You can use exponents to show repeated multiplication. For example, you can write \(3 \times 3 \times 3 \times 3\) as \(3^4\). You also can express repeated multiplication of rational numbers using exponents.

\[
\left(\frac{1}{2}\right)^5 = \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} = \frac{1}{32}
\]

**Getting Ready for Problem 1.1**

The rules of exponents allow you to rewrite some expressions. You can simplify an expression like \(4^2 \times 4^3\), because \(4^2\) and \(4^3\) have the same base.

\[
4^2 \times 4^3 = (4 \times 4) \times (4 \times 4 \times 4) = 4 \times 4 \times 4 \times 4 \times 4 = 4^5
\]

Notice that \(2 + 3 = 5\). To multiply two numbers with the same base, add the exponents.

You can use the rule for multiplying two numbers with the same base to simplify an expression like \((6^5)^2\).

\[
(6^5)^2 = 6^5 \times 6^5 = 6^{(5+5)} = 6^{10}
\]

Notice that \(5 \times 2 = 10\). To raise a power to a power, multiply the exponents.

The rules of exponents also allow you to simplify expressions involving division. To divide two numbers with the same base, subtract the exponents.

\[
\frac{9^5}{9^3} = 9^{(5-3)} = 9^2
\]

* How can you write the expression \(7^5 \times 7\) using a single exponent?
* How can you write the expression \((12^5)^{11}\) using a single exponent?
* How can you write the expression \(8^7 \div 8^3\) using a single exponent?

Negative exponents can be used to represent repeated division.
A. You can use patterns to find the values of negative exponents.

1. Find the values of $2^4$, $2^3$, and $2^2$.
2. What pattern do you see as you go from $2^4$ to $2^3$ and from $2^3$ to $2^2$?
3. The value of $2^5$ is 256. Show how to find the value of $2^7$ using division.

B. 1. Extend the pattern you found in Part A. Find $2^1$, $2^0$, $2^{-1}$, $2^{-2}$, $2^{-3}$, and $2^{-4}$. Express values less than 1 as fractions.
2. You can use multiplicative inverses (reciprocals) to find the value of an expression with a negative exponent. Complete each statement.

$$2^{-2} = \left( \frac{1}{2} \right)^2$$
$$2^{-3} = \left( \frac{1}{2} \right)^3$$
$$2^{-4} = \left( \frac{1}{2} \right)^4$$

3. Rewrite the expression using its reciprocal and a positive exponent. Then find the value of the expression.

a. $4^{-2}$

b. $\left( \frac{2}{3} \right)^{-3}$

Finding a number when you know its cube is called finding the **cube root**. This is shown with the $\sqrt[3]{\text{•}}$ symbol. For example, $\sqrt[3]{8} = 2$ because $2^3 = 8$.

### Problem 1.2

A. Find the values of $\sqrt[3]{27}$ and $\sqrt[3]{125}$.

B. 1. The formula $V = e^3$ represents the volume, $V$, of a cube with edge length $e$. Explain how you can use the formula to find the edge length of a cube if you are given its volume.

2. A company sells small gift boxes with the volumes shown in the table. Copy and complete the table, using the formula for the volume of a cube to find each box’s edge length.

<table>
<thead>
<tr>
<th>Volume (in.$^3$)</th>
<th>Edge Length (in.)</th>
</tr>
</thead>
<tbody>
<tr>
<td>8</td>
<td></td>
</tr>
<tr>
<td>64</td>
<td></td>
</tr>
<tr>
<td>343</td>
<td></td>
</tr>
<tr>
<td>729</td>
<td></td>
</tr>
</tbody>
</table>

3. A vase in the shape of a cube will hold 384 cubic inches of water when filled three-quarters of the way to the top. What is the length of an edge of the vase?
Exercises

For Exercises 1–3, write each expression using an exponent.

1. \(3.6 \times 3.6 \times 3.6 \times 3.6\)
2. \(\frac{4}{5} \times \frac{4}{5} \times \frac{4}{5}\)
3. \(0.8 \times 0.8 \times 0.8 \times 0.8 \times 0.8\)

For Exercises 4–9, find the value of each expression.

4. \(\left(\frac{1}{3}\right)^5\)
5. \((3.6)^2\)
6. \((0.4)^3\)
7. \((2.1)^3 \times (3.5)^2\)
8. \(\left(\frac{2}{3}\right)^3 \times \left(\frac{3}{2}\right)^2\)
9. \((0.2)^4 \times \left(\frac{3}{2}\right)^2\)

10. For parts a–d, find the value of each expression.
   a. \((0.1)^1\)
   b. \((0.1)^2\)
   c. \((0.1)^3\)
   d. \((0.1)^4\)
   e. Describe the pattern in parts a–d for the exponent and the value of the corresponding expression.
   f. Use the pattern you described to write the value of \((0.1)^8\).

For Exercises 11–16, write each expression using a single exponent.

11. \(7^2 \times 7^{13}\)
12. \((36)^7\)
13. \(235^{141} \times 235^{37}\)
14. \((17^4)^{20}\)
15. \(5^3 \times 5^9 \times 5^{16}\)
16. \(((2^4)^2)^5\)

The rules of exponents also apply to expressions with variables. For Exercises 17–19, write each expression using a single exponent.

17. \((x^5)(x^7)\)
18. \(\frac{m^{13}}{m^6}\)
19. \((p^8)^3\)

For Exercises 20–31, find the value of each expression.

20. \((2)^{-1}\)
21. \((3)^{-5}\)
22. \((5)^{-3}\)
23. \(\left(\frac{2}{3}\right)^{-1}\)
24. \(\left(\frac{8}{3}\right)^{-3}\)
25. \(\left(\frac{7}{16}\right)^{-2}\)
26. \(2 \times (4)^{-2}\)
27. \(\left(\frac{1}{5}\right)^{-3} \times \frac{4}{5}\)
28. \(\left(\frac{1}{2}\right)^{-4} \times \left(\frac{2}{3}\right)^{-2}\)
29. \((5)^{-2} \times (10)^2\)
30. \((5)^{-3} \times \left(\frac{2}{5}\right)^{-4}\)
31. \(\left(\frac{3}{8}\right)^{-3} \times \left(\frac{3}{2}\right)^{-4}\)

For Exercises 32–37, find the value of each expression. Write values less than 1 using decimals.

32. \(2 \times (10)^2\)
33. \(2 \times (10)^{-2}\)
34. \(7 \times (10)^{-3}\)
35. \(7 \times (10)^3\)
36. \(2.5 \times (10)^5\)
37. \(2.5 \times (10)^{-5}\)
38. Anthony and Alexandra found the value of \((6)^3 \times (3)^{-2}\) in different ways. Are both methods correct? Why?

Anthony’s Method:
I will rewrite using positive exponents. Then I will simplify the powers and multiply.
\[
(6)^3 \times (3)^{-2} = (6)^3 \times \left(\frac{1}{3}\right)^2
\]
\[
= 216 \times \frac{1}{9}
\]
\[
= 24
\]

Alexandra’s Method:
I will write positive exponents as repeated multiplication and negative exponents as repeated division. Then I will perform the operations.
\[
(6)^3 \times (3)^{-2} = \left[6 \times 6 \times 6\right] \div 3 \div 3
\]
\[
= 216 \div 3 \div 3
\]
\[
= 72 \div 3
\]
\[
= 24
\]

39. For parts a and b, find the value of each expression.
   a. \(5^4 \quad 5^3 \quad 5^2 \quad 5^1 \quad 5^0\)
   b. \(10^4 \quad 10^3 \quad 10^2 \quad 10^1 \quad 10^0\)
   c. Look for a pattern in your answers to parts a and b. Use the pattern to predict the value of any positive number with exponent zero.

For Exercises 40–45, find the value of each cube root.

40. \(\sqrt[3]{1}\) \(\sqrt[3]{64}\)
41. \(\sqrt[3]{216}\) \(\sqrt[3]{125}\)
42. \(\sqrt[3]{1,000}\) \(\sqrt[3]{343}\)

For Exercises 46–51, solve for \(x\).

46. \(x^3 = 125\)
47. \(x^3 = 27\)
48. \(x^3 = 216\)
49. \(x^3 = 1,000\)
50. \(x^3 = 1\)
51. \(x^3 = 64\)

52. A cubic shipping carton has a volume of 343 cubic inches. What is the length of an edge of this carton?
CC Investigation 2: Functions

Mathematical Goals

- Graph proportional relationships, interpreting the unit rate as the slope of the graph.
- Compare proportional relationships represented in different ways.
- Use similar triangles on the coordinate plane to explain why the slope between any two points on a line is a constant.
- Derive the equations \( y = mx \) and \( y = mx + b \) to describe lines on the coordinate plane.
- Simplify a linear equation in one variable to determine whether it has no solution, one solution, or infinitely many solutions.
- Understand that a function is a rule that assigns a unique output to each input, and that a graph of a function is a set of ordered pairs consisting of each input and corresponding output.

Teaching Notes

This Investigation reviews concepts developed in the Grade 7 Units Variables and Patterns and Moving Straight Ahead. Check for prior knowledge before beginning this Investigation.

In this investigation, students will study proportions in the form \( y = mx \) and recognize that they are special linear equations \( (y = mx + b) \), where the constant of proportionality, \( m \), is the slope, and the graphs are lines that pass through the origin.

Students will work with functions in different representations, including equations, graphs, and tables. To give students practice in translating among different representations, present them with the equation \( y = 3x \). Have students complete this table using the equation.

<table>
<thead>
<tr>
<th>( x )</th>
<th>0</th>
<th>1</th>
<th>3</th>
<th>5</th>
<th>6</th>
</tr>
</thead>
<tbody>
<tr>
<td>( y )</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Guide students to see that each related pair of \( x \)- and \( y \)-values makes up a coordinate pair that can be graphed on the coordinate plane. Have students graph the equation.
**Problem 2.1**

*Before Problem 2.1, review the definition and representation of proportional relationships. Ask:*

- How do you know that the graph of an equation in the form \( y = kx \) will be a straight line? (For each increase in \( x \) there is a proportional increase in \( y \).)

- What will the corresponding \( y \)-value be for an \( x \)-value of 0 in any proportion? (0)

*During Problem 2.1 A, ask:*

- How do you find a unit rate for speed? (Divide distance by time to find the distance covered in 1 unit of time.)

- Does it matter what points you use to find the unit rate for speed? (No, because the graph is a line, any two points on the line will give the same unit rate.)

*During Problem 2.1 C, ask: What expression can you write using the first two data pairs in the table to find the unit rate for Carlos’s speed? \( \frac{4 - 2}{19 - 9.5} \)

**Problem 2.2**

*Before Problem 2.2 A, confirm that students are familiar with basic geometric properties of similarity and angles created by transversals of parallel lines. Present this figure to students.*

*Ask:*

- How are the angles and sides of similar plane figures related? (The angles all are equal and the side lengths are proportional.)

- If lines \( j \) and \( k \) are parallel, how are angles 2 and 6 related? (They are corresponding angles and their measures are equal.)

- If line \( t \) were perpendicular to lines \( j \) and \( k \), what would the measures of angles 2 and 6 be? (90°)

*During Problem 2.2 A Part 4, ask: How is the line of the graph related to triangle sides \( BD \) and \( AE \)? (It is a transversal of two parallel segments.)*

*During Problem 2.2 B, ask:*

- How are the triangles in this part like the triangles in Part A? (The hypotenuses lie along the same line, and the other corresponding sides are parallel.)

- What does the fact that sides \( a \) and \( d \) lie on the same line tell you about their slopes? (They are equal.)

*During Problem 2.2 C, ask: How is Ricardo’s 10 m head-start represented on the graph? (His distance at \( t = 0 \) is 10 m.)
Problem 2.3

• Before Problem 2.3, remind students about the properties of equality and how they must be followed to simplify an equation. Ask:
  • If I divide one side of an equation by 4, what must I do to the other side of the equation so that the equation will remain true? (Divide it by 4.)
  • If I subtract $6t$ from one side of an equation, what must I do to the other side of the equation so that the equation will remain true? (Subtract $6t$.)

During Problem 2.3, ask: How do you represent a situation where one runner will pass another? (Set their distances in the race equal for the same time.)

Problem 2.4

Before Problem 2.4, review the definition of a function, and draw students’ attention to the 50-meter Dash line graphs in Problems 2.1 and 2.2. Ask:

• Are these graphs of functions? How do you know? (Yes; there is only one y-value for any given x-value.)
  • What general rule can you take from these graphs about graphs of functions? (A graph that is a straight line represents the graph of a function.)

During Problem 2.4 B, ask: How is the graph of the equation $d = 15t$ similar to the graphs in Problems 2.1 and 2.2? (They all are straight lines.)

After Problem 2.4, ask: How would the graph of an equation that is not a function look? (There would be 2 or more y-values for a given x-value. The graph could look like a sideways V or U.)

Summarize

To summarize the lesson, ask:

• How can you find the unit rate when given a graph of a proportional relationship? (The unit rate is the slope of the graph.)
  • How can you find the unit rate of a proportional relationship shown in a table of x- and y-values? (Divide the change in any two y-values by the corresponding change in x-values.)
  • What do you know about the right triangles formed using any two points along the graph of a function? (They are similar.)
  • How many outputs can be assigned to each input for a function? (only one)
  • How can you determine how many solutions a linear equation with one variable will have? (Simplify the equation until it is in the form $x = a$, $a = a$, or $a = b$, where $a$ and $b$ represent two different real numbers.)
**Assignment Guide for Investigation 2**

Problem 2.1, Exercises 1–4  
Problem 2.2, Exercises 5  
Problem 2.3, Exercises 6–13  
Problem 2.4, Exercises 14–22

**Answers to Investigation 2**

**Problem 2.1**

A. 1. The graph is a straight line, so Ramon ran at a constant speed during the race.

2. The distance that corresponds to 1 second is 5 meters, so Ramon ran at a speed of 5 m/sec.

B. For equations in the form $y = mx$, the constant $m$ represents the unit rate. Here that rate is 4.5, so Angel ran at a speed of 4.5 m/sec.

C. 1. [Graph of 50-meter Dash]

2. Divide the distance at any point on the graph by the corresponding time: $9.5 \div 2 = 4.75$, so Carlos ran at a speed of 4.75 m/sec.

D. Ramon won; his speed was the greatest: $5 > 4.75 > 4.5$.

**Problem 2.2**

A. 1. each angle measures $90^\circ$

2. $\angle C$

3. Since two pairs of angles are congruent, the third pair is also congruent. Since all corresponding angles are congruent, the triangles are similar.

4. a. They are corresponding angles of similar triangles, so they are congruent.  
   b. The speed was the same since the slopes of the corresponding sides of both triangles are the same.

B. 1. The triangles are similar. Since sides $a$ and $d$ lie along the same line, the angle between sides $a$ and $c$ is congruent to the angle between sides $d$ and $f$. The right angles of the triangles also are congruent, so the triangles are similar.

2. Similar triangles can be drawn using any two points along the line. Because they are similar, the ratios of the lengths of the corresponding sides are equivalent, so the slopes, which are the ratios of the vertical side lengths to the horizontal side lengths, also are equivalent.

C. 1. $y = 5x$

2. Ramon’s and Ricardo’s starting positions in the race.

3. Yes; because the speeds are equal, the slopes are equal, and the lines are parallel.

4. They are the same.

5. $y = 5x + 10$

6. $y = mx + b$

**Problem 2.3**

A. 1. $4t = 5t - 7$

2. $4t = 5t - 7; 4t - 5t = 5t - 7 - 5t; -t = -7; t = 7$

There is one solution.

3. Micah will catch Ramon 7 seconds after the start of the race.

B. 1. $4t = 4t - 14.7$

2. $4t = 4t - 14.7; 4t - 4t = 4t - 14.7 - 4t; 0 = -14.7; There are no solutions.$

3. Joel will not catch Ramon in this race.
C. 1. \(4t = 4t\)
2. \(4t = 4t; 4t \div t = 4t \div t; 4 = 4\); There are an infinite number of solutions.
3. Ben and Ramon will be at the same distance throughout the entire race.

Problem 2.4
A.

<table>
<thead>
<tr>
<th>Time, (t) (hours)</th>
<th>Distance, (d) (miles)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.5</td>
<td>7.5</td>
</tr>
<tr>
<td>1.5</td>
<td>22.5</td>
</tr>
<tr>
<td>1.75</td>
<td>26.25</td>
</tr>
<tr>
<td>2</td>
<td>30</td>
</tr>
</tbody>
</table>

B. 1. \((0.5, 7.5), (1.5, 22.5), (1.75, 26.25), (2, 30)\)
2. Sample: The point \((1, 15)\) also lies on the graph, and represents the distance, 15 miles, that Ramon can ride in 1 hour.

Exercises
1. Pool A: 4 gal/min; Pool B: 3 gal/min
2. Pool B: Pool B filled at a constant rate of 15 gallons every 5 minutes, so it would have had \(22 - 15 = 7\) gallons of water in it at 0 minutes.
3. Pool A: \(y = 4x\); Pool B: \(y = 3x + 7\)

5. a and b Sample:
6. A
7. C
8. one solution
9. infinitely many solutions
10. no solutions
11. infinitely many solutions
12. no solutions
13. one solution
14. | x | y |
<table>
<thead>
<tr>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>-1</td>
</tr>
<tr>
<td>2</td>
<td>1</td>
</tr>
<tr>
<td>3</td>
<td>3</td>
</tr>
<tr>
<td>4</td>
<td>5</td>
</tr>
</tbody>
</table>
15. | x  | y  |
    |---|---|
    | -2| -6 |
    | -1| -1 |
    | 0 |  4 |
    | 1 |  9 |
16. | x  | y   |
    |---|-----|
    | -2| 11  |
    | 0 | 12  |
    | 2 | 13  |
    | 4 | 14  |
17. Yes, there is only one y-value for each x-value.
18. No, there is more than one y-value for some x-values.
19. No, there is more than one y-value for the x-value of 2.
20. Yes, there is only one y-value for each x-value.
21. No, there is more than one y-value for the x-value of -2.
22. a. | Pace, s (in min/mi) | Finishing Time, t (in min) |
     |---|-------------------|
     | 8 | 189.6             |
     | 9 | 195.8             |
     | 10| 202               |
     | 12| 214.4             |
23. b. (8, 189.6), (9, 195.8), (10, 202), (12, 214.4)
24. c. one; The equation \( t = 140 + 6.2s \) is a function.
The graph shows the positions of two trains on the same track.

1. What do the shapes of the graphs tell you about the trains’ speeds?

2. Explain how you can use the graph to find train B’s speed.

3. What do the points (0, 0) and (0, 60) on the graph represent?

4. Represent train B’s position with an equation.

5. Represent train A’s position with an equation.

6. Explain how to use the equation you wrote in Exercise 5 to identify train A’s speed.

7. Look at the graph for train A. How many distances are there for each time? What does this tell you about the equation for train A’s position?

8. Train C leaves Omaha on the same track, and its position can be represented by the expression $75(t - 1)$.
   a. Write an equation by setting the expressions representing the positions of trains B and C equal to each other.
   b. What does the equation represent?
   c. Solve the equation, if possible. How many solutions are there?
   d. What does the solution tell you about the positions of trains B and C?
Skill: Finding Unit Rate

Find the unit rate.

1. \[ \text{Distance (ft)} \]
   \[ \text{Time (sec)} \]
   \[
   \begin{array}{l}
   0 \quad 1 \quad 2 \quad 3 \\
   0 \quad 1 \quad 2 \quad 3 \\
   \end{array}
   \]

2. \[ \text{Volume of Water (L)} \]
   \[ \text{Time (min)} \]
   \[
   \begin{array}{l}
   0 \quad 1 \quad 2 \quad 3 \quad 4 \quad 5 \\
   0 \quad 1 \quad 2 \quad 3 \quad 4 \quad 5 \\
   \end{array}
   \]

3. | Weight (lb) | 2 | 4 | 5 | 9 |
   | Price ($)  | 5 | 10 | 12.50 | 22.50 |

4. | Height (cm) | 9 | 36 | 42 | 60 |
   | Time (day)  | 3 | 12 | 14 | 20 |

5. | Distance (mi) | 150 | 210 | 450 | 540 |
   | Time (h)    | 2.5 | 3.5 | 7.5 | 9 |

6. | Temperature (°F) | 148 | 296 | 370 | 481 |
   | Time (min)   | 4 | 8 | 10 | 13 |

7. \[ y = 14.5x \]

8. \[ y = \frac{x}{2} \]

Skill: Solutions to Equations

Tell whether the equation has one solution, no solutions, or infinitely-many solutions.

9. \[ 3t = 5t - 10 \]
10. \[ 3x + 5 = 3x - 5 \]
11. \[ 8r + 4 = 8r - 2 \]
12. \[ g - 6 = 4g \]
13. \[ 2d + 7 = d + 7 \]
14. \[ 5h = h(4 + 1) \]
15. \[ w - 4.8 = 94 + w \]
16. \[ 4g + 2 = 2g + 2 + 2g \]
17. \[ 4 - 15a = a \]
18. \[ 6(t - 2) = 6t - 12 \]
1. The graph shows the amount of water in a gallon jug that Catelyn filled from a hose.

[Graph of Water in a Jug]

a. Explain how you can use the graph to find the unit rate at which water filled the jug.

b. Catelyn turned the spigot on the hose and filled a second jug. The volume of water in that jug can be represented by the equation \( v = 25t \). Explain how to find the unit rate from the equation.

c. Catelyn turned the spigot again and filled a third jug. The table shows the volume of water in that jug at different times. Explain how you can find the unit rate from the table.

<table>
<thead>
<tr>
<th>Time (sec)</th>
<th>2</th>
<th>3.5</th>
<th>5</th>
<th>6.5</th>
</tr>
</thead>
<tbody>
<tr>
<td>Volume (fl oz)</td>
<td>32</td>
<td>56</td>
<td>80</td>
<td>104</td>
</tr>
</tbody>
</table>

d. Which jug did Catelyn fill the fastest? Explain how you know.
A relationship between two quantities is **proportional** if the ratio between the quantities is always the same unit rate. Proportional relationships can be represented by the equation \( y = kx \), where \( k \) represents a constant. The graph of any proportional relationship will be a straight line through the origin.

**Problem 2.1**

Ramon raced Angel and Carlos in a 50-meter dash.

**A.** Ramon’s results are shown on the graph.
   1. What does the shape of the graph tell you about Ramon’s speed during the race?
   2. Explain how you can use the graph to find the unit rate for Ramon’s speed.

**B.** Angel’s data during the race can be described using the equation \( y = 4.5x \). Explain how you can find the unit rate for Angel’s speed from the equation.

**C.** Carlos ran the race at a constant speed. The table shows the distances Carlos traveled during different times in the race.

<table>
<thead>
<tr>
<th>Time (in sec)</th>
<th>2</th>
<th>4</th>
<th>6</th>
<th>8</th>
</tr>
</thead>
<tbody>
<tr>
<td>Distance (in m)</td>
<td>9.5</td>
<td>19</td>
<td>28.5</td>
<td>38</td>
</tr>
</tbody>
</table>

1. Plot the data on a graph to show Carlos’s speed during the race.
2. Explain how you can use the graph to find the unit rate for Carlos’s speed.

**D.** Who won the race? Explain how you know.
A. Ramon drew triangles \( ACE \) and \( BCD \) on the graph of his race times. Triangle sides \( BD \) and \( AE \) are parallel to the \( x \)-axis and side \( CE \) is parallel to the \( y \)-axis.

1. What are the measures of \( \angle AEC \) and \( \angle BDC \)?
2. What angle do triangles \( ACE \) and \( BCD \) share?
3. What do your answers in Parts 1 and 2 tell you about how triangles \( ACE \) and \( BCD \) are related?
4. a. How are \( \angle CAE \) and \( \angle CBD \) related?
   b. What does that relation tell you about Ramon’s speed from 2 sec to 5 sec in the race compared to his speed from 2 sec to 8 sec in the race?

B. Look at the triangles. Sides \( a \) and \( d \) lie along the same line.

1. How are the triangles related to each other? Explain how you know.
2. Use the graph to explain why the slope \( m \) is the same between any two points along the line.
C. Suppose Ramon’s twin brother, Ricardo, also runs in the race. Ramon gives Ricardo a 10-m head-start in the race, and they run at the same speed. The graph shows their results.

![Graph showing distance vs. time for Ramon and Ricardo.]

1. Represent Ramon’s position with an equation.
2. What do the points (0, 0) and (0, 10) on the graph represent?
3. Are the lines parallel? Explain.
4. How does a translation of Ramon’s line 10 units up compare with Ricardo’s line on the graph?
5. Ricardo runs at a constant rate of 5 m/sec and has a head start of 10 m. Write an equation of the line representing Ricardo’s position, \( y \), at time \( x \).
6. Write an equation in slope-intercept form for a line that has slope \( m \) and contains the point \( (0, b) \).

Linear equations with one variable can have one solution, infinitely many solutions, or no solutions. Use properties of operations to balance an equation until you get one of these forms, where \( a \) and \( b \) represent two different numbers.

<table>
<thead>
<tr>
<th>Simplest Form of Equation</th>
<th>Number of Solutions</th>
</tr>
</thead>
<tbody>
<tr>
<td>( x = a )</td>
<td>one</td>
</tr>
<tr>
<td>( a = a )</td>
<td>infinite</td>
</tr>
<tr>
<td>( a = b )</td>
<td>none</td>
</tr>
</tbody>
</table>

Notes
For a practice race, the track coach starts Micah and Joel different distances behind Ramon and Ben. The expression $4t$ shows Ramon’s distance $t$ seconds after the start of the race.

A. The expression $5t - 7$ shows Micah’s distance $t$ seconds after the start of the race.
   1. Write an equation by setting the expressions equal to each other to represent the point where Micah could catch Ramon in the race.
   2. Solve the equation, if possible. How many solutions are there?
   3. What does the simplified equation tell you about whether Micah will catch Ramon?

B. The expression $4t - 14.7$ shows Joel’s distance $t$ seconds after the start of the race.
   1. Write an equation by setting the expressions equal to each other to represent the point where Joel could catch Ramon in the race.
   2. Solve the equation, if possible. How many solutions are there?
   3. What does the simplified equation tell you about whether Joel will catch Ramon?

C. The expression $4t$ shows Ben’s distance $t$ seconds after the start of the race.
   1. Write an equation by setting the expressions equal to each other to represent a point where Ben and Ramon could be at the same distance in the race.
   2. Solve the equation, if possible. How many solutions are there?
   3. What does the simplified equation tell you about when Ramon and Ben will be at the same distance?
2. Michael, Marta, Jake, and Ellie take their canoes to a lake. Each paddles away from shore at a constant speed.
   
a. The equation $d = 5t$ represents the distance in feet, $d$, Michael is away from shore after $t$ seconds. Complete the table to show the distances Michael is away from shore after different amounts of time.

<table>
<thead>
<tr>
<th>Time, $t$ (sec)</th>
<th>Distance, $d$ (ft)</th>
</tr>
</thead>
<tbody>
<tr>
<td>3</td>
<td></td>
</tr>
<tr>
<td>4.5</td>
<td></td>
</tr>
<tr>
<td>7</td>
<td></td>
</tr>
<tr>
<td>10.5</td>
<td></td>
</tr>
</tbody>
</table>

b. How many related distances are there for each of the times in the table? What does this tell you about the equation $d = 5t$?

c. Marta leaves the shore in her canoe 5 seconds after Michael, and paddles in the same direction. The expression $6(t - 5)$ shows Marta’s distance from shore $t$ seconds after Michael left. Will Marta be able to catch Michael? Explain how you know, and show your work.

d. Jake leaves the shore in his canoe 8 seconds after Michael, and paddles in the same direction. The expression $4.5(t - 8)$ shows Jake’s distance from shore $t$ seconds after Michael left. Will Jake be able to catch Michael? Explain how you know, and show your work.

e. Ellie leaves the shore at the same time as Jake, and she paddles in the same direction at a constant rate of 4.5 ft/sec. Describe Jake’s and Ellie’s positions relative to each other for the first 10 seconds that they are paddling. Explain your reasoning.
A function is a relationship that pairs each input value with exactly one output value.

**Problem 2.4**

Ramon is riding his bike to get himself more fit for track season. For training, he rides his bike at a constant speed of 15 mi/h. The equation \( d = 15t \) shows the number of miles, \( d \), that Ramon can ride in \( t \) hours.

**A.** Copy and complete the table to show the distances Ramon can ride in different amounts of time.

<table>
<thead>
<tr>
<th>Time, ( t ) (hours)</th>
<th>Distance, ( d ) (miles)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.5</td>
<td></td>
</tr>
<tr>
<td>1.5</td>
<td></td>
</tr>
<tr>
<td>1.75</td>
<td></td>
</tr>
<tr>
<td>2</td>
<td></td>
</tr>
</tbody>
</table>

**B. 1.** List the ordered pairs represented in the table.

**2.** Graph the ordered pairs on a coordinate grid. Connect the points to show the solution to the equation \( d = 15t \). What do you notice about the shape the data points take?

**3.** Give the ordered pair for another point on the graph. Explain what that point represents.

**4.** How many different values of \( d \) are possible for each value of \( t \)? What does this tell you about the equation \( d = 15t \)?
Exercises

For Exercises 1–4, use the tables below, which show the amounts of water in two pools as they are filled. Each pool is filled at a constant rate.

<table>
<thead>
<tr>
<th>Minutes</th>
<th>Gallons</th>
<th>Minutes</th>
<th>Gallons</th>
</tr>
</thead>
<tbody>
<tr>
<td>5</td>
<td>20</td>
<td>5</td>
<td>22</td>
</tr>
<tr>
<td>10</td>
<td>40</td>
<td>10</td>
<td>37</td>
</tr>
<tr>
<td>15</td>
<td>60</td>
<td>15</td>
<td>52</td>
</tr>
<tr>
<td>20</td>
<td>80</td>
<td>20</td>
<td>67</td>
</tr>
</tbody>
</table>

1. Write the filling rate for each pool in gallons per minute.
2. One of the pools was not empty when it started to be filled. From the data in the tables, which pool was not empty? Explain how you know.
3. Write a linear equation in slope-intercept form for each set of data.
4. Graph the equations on the same coordinate plane.
5. The graph shows the temperature in an oven as it heats up.

a. Choose two points on the graph and draw dotted lines between them to show the rise and run.

b. Use one of the points from part a and another point on the line. Draw dotted lines to represent the rise and run between these points.

c. Describe the relationship between the two triangles formed by the lines, and the slopes of the two sets of points.

Notes
6. **Multiple Choice** What is an equivalent form of $x + 5 = 2x - x + 5$?
   A. $x = x$
   B. $5x + 5 = x$
   C. $x + 5 = x - 5$
   D. $2x + 5 = 2x - 5$

7. **Multiple Choice** What is an equivalent form of $y + 5 = 2y - y - 5$?
   A. $y = y$
   B. $y = y - 5$
   C. $5 = -5$
   D. $y + 10 = 2y + 5$

For Exercises 8–13, tell whether the equation has **one solution, no solutions, or infinitely-many solutions**.

8. $x + 21 = 101$  
9. $t + 5 = t + 5$
10. $a = a + 12$  
11. $2w + 21 = 5w - 3w + 21$
12. $9k + 1 = 4k - 1 + 5k$  
13. $37 = m - 9$

For Exercises 14–16, copy and complete the table. Then graph the function.

14. $y = 2x - 3$  
15. $y = 5x + 4$  
16. $y = \frac{1}{2}x + 12$

<table>
<thead>
<tr>
<th>$x$</th>
<th>$y$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td></td>
</tr>
<tr>
<td>2</td>
<td></td>
</tr>
<tr>
<td>3</td>
<td></td>
</tr>
<tr>
<td>4</td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>$x$</th>
<th>$y$</th>
</tr>
</thead>
<tbody>
<tr>
<td>-2</td>
<td></td>
</tr>
<tr>
<td>-1</td>
<td></td>
</tr>
<tr>
<td>0</td>
<td></td>
</tr>
<tr>
<td>1</td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>$x$</th>
<th>$y$</th>
</tr>
</thead>
<tbody>
<tr>
<td>-2</td>
<td></td>
</tr>
<tr>
<td>0</td>
<td></td>
</tr>
<tr>
<td>2</td>
<td></td>
</tr>
<tr>
<td>4</td>
<td></td>
</tr>
</tbody>
</table>
For Exercises 17–21, explain whether the data shown represent a function.

17. **y**

18. **y**

19. | x  | y  |
<table>
<thead>
<tr>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>6</td>
</tr>
<tr>
<td>2</td>
<td>9</td>
</tr>
<tr>
<td>4</td>
<td>12</td>
</tr>
<tr>
<td>5</td>
<td>18</td>
</tr>
</tbody>
</table>

20. | x  | y  |
<table>
<thead>
<tr>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>7</td>
</tr>
<tr>
<td>4</td>
<td>13</td>
</tr>
<tr>
<td>6</td>
<td>19</td>
</tr>
<tr>
<td>8</td>
<td>25</td>
</tr>
</tbody>
</table>

21. | x  | y  |
<table>
<thead>
<tr>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>-2</td>
<td>1</td>
</tr>
<tr>
<td>0</td>
<td>5</td>
</tr>
<tr>
<td>-2</td>
<td>2</td>
</tr>
<tr>
<td>2</td>
<td>9</td>
</tr>
</tbody>
</table>

22. Ramon has completed the swimming and biking portions of a triathlon in a total time of 140 minutes. The equation \( t = 140 + 6.2s \) shows Ramon’s finishing time, \( t \), if he runs at an average pace of \( s \) minutes per mile.

   a. Copy and complete the table to show Ramon’s finishing times for different running paces.

<table>
<thead>
<tr>
<th>Pace, ( s ) (in min/mi)</th>
<th>Finishing Time, ( t ) (in min)</th>
</tr>
</thead>
<tbody>
<tr>
<td>8</td>
<td></td>
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<td>9</td>
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<td>10</td>
<td></td>
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<tr>
<td>12</td>
<td></td>
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</tbody>
</table>

   b. List the ordered pairs represented in the table. Graph the ordered pairs and connect them with a line to show the solutions to the equation.

   c. How many different values of \( t \) are possible for each value of \( s \)?
   What does this tell you about the equation \( t = 140 + 6.2s \)?
CC Investigation 3: Transformations

Mathematical Goals

- Verify the properties of translations, rotations, and reflections of figures including lines, line segments, angles, and parallel lines.

- Describe the effects of translations, rotations, reflections, and dilations on two-dimensional figures using coordinates.

- Understand that two figures are similar if one can be made from the other by a sequence of translations, rotations, reflections, or dilations.

- Describe the sequence of translations, rotations, reflections, or dilations that exhibits the similarity of two similar figures.

Teaching Notes

Among the transformations your students will study—translations, rotations, reflections, and dilations—translations should be the easiest for the students to grasp. Students will find translations relatively easy to model by tracing the outline of a flat shape on graph paper before and after sliding the shape from one location to another. In those activities where students copy a shape and then draw its prescribed translation, tracing paper can be used to confirm that the original shape and the translated image are congruent.

In the problems and exercises, the points being used for the translation are all vertices. You should briefly review the definition of a vertex as the point in a polygon where two sides meet.

Once students understand the reflection of a point across a line, the notion of reflecting a figure across a line by reflecting vertices and then connecting them should come naturally. Although Problem 3.2 involves figures in the first quadrant, you should use your judgment with regard to presenting examples of reflections across the $x$- and $y$-axes. If you do so, remind students to find reflected points simply by counting units between points and the line of reflection.

Rotations can be more problematic for students than translations or reflections. Even the $180^\circ$ rotation of a shape about the origin, as presented in Problem 3.3 can present difficulties as compared with a reflection of the same image across the $x$-axis. It may help students to concentrate on one vertex as a “leading point,” visualize the arc made by that point, and then concentrate on the “trailing points” that follow the leader along the arcs of concentric circles.

No matter where the center of rotation is located—inside, outside or on the figure—the key indicator of a rotation is the distance from the center of rotation to any particular point on the figure. To confirm that a rotation has occurred, students can compare pre- and post-rotation distances of a given point with a compass, a ruler, or the marked edge of a piece of paper.

Vocabulary

- transformation
- translation
- reflection
- line of reflection
- rotation
- center of rotation
- clockwise
- counterclockwise
- dilation
- scale factor
- center of dilation

Materials

- graph paper
A dilation is the enlargement or reduction of a figure. The size of the figure changes but the shape does not, so the original figure and the dilation are similar. The scale factor used in a dilation determines if the dilation is larger or smaller than the original, and how much larger or smaller it is. The center of dilation can be any point on the coordinate plane that is inside, on, or outside the original figure.

In Problem 3.7, students will use what they know about translations, rotations, reflections, and dilations to explain how one of a pair of similar figures can be transformed into the other. Remind students that any transformation will result in a figure that is similar to the original.

**Problem 3.1**

**Before** Problem 3.1, ask: *What about the figures in the design changes as you move from left to right in the design?* (only their positions).

**During** Problem 3.1, ask:
- Why do you think it is a good idea to name the translated triangle with \( A', B', \) and \( C' \) instead of using other letters? (It makes it easier to match up the original vertices to the translated vertices.)
- What has changed about \( \triangle ABC \) once you translated it to be \( \triangle A'B'C' \)? (only its position)
- How could you make sure that \( \triangle ABC \) is the same size and shape as \( \triangle A'B'C' \)? (Trace one triangle onto tracing paper and see if they match.)

**Problem 3.2**

**After** Problem 3.2 B, ask: *How could you draw the reflection of a line segment across a line of reflection?* (Reflect the segment’s endpoints and then connect the two reflected points.)

**After** Problem 3.2 C ask:
- Does the line of reflection have to be touching a figure and its reflected image? (no)
- How far away from a line of reflection can a figure and its reflected image be? (There is no mathematical limit.)
- What is the procedure for drawing the reflection of a triangle across a line that runs through the triangle? (It is the same procedure as for a line of reflection exterior to the triangle: reflect the vertices across the line, then connect the reflected vertices.)

**Problem 3.3**

**Before** Problem 3.3, ask:
- How many degrees of a counterclockwise rotation has the same effect as a \( 270° \) clockwise rotation? (90°)
- What part of the Ferris wheel at the top of the page does not change its location no matter what type of rotation takes place? (the center)

**After** Problem 3.3, ask:
- How can you tell that \( K'L'M'O \) is not a reflection of \( KLMO \) across the \( x \)-axis? (Corresponding points are not the same distance away from the \( x \)-axis on the opposite side.)
- How would the graph look different if the \( 180° \) rotation had been counterclockwise instead of clockwise? (There would be no difference.)
Problem 3.4

Before Problem 3.4, ask: How is this rotation different than the rotation in Problem 3.3? (The point of rotation is outside the figure in this problem, instead of at one of the figure’s vertices, as in Problem 3.3.)

After Problem 3.4 C, ask: If you draw lines from other corresponding vertices of the triangles to the center of rotation, what will the angle between the lines be? (90°)

Problem 3.5

After Problem 3.5, ask:

- If the dilation had instead been from ΔA′OB′ to ΔAOB, would it have been an enlargement or reduction? (reduction)
- What would the scale factor have been? \( \left( \frac{1}{2} \right) \)
- What general rule can you write to relate the new scale factor if a dilation is reversed to the original scale factor? (Reverse scale factors are reciprocals of each other.)

Problem 3.6

After Problem 3.6, ask:

- Could you discover the center of dilation by just looking at the figures? (Yes, the center of each figure is at (2, 2), which is the center of dilation.)
- What would the scale factor be if figure K′L′M′N′ were dilated to make figure KLMN? (4)
- Does the location of the center of dilation affect the size of the dilated figure? (No, it affects only the location of the figure.)

Problem 3.7

Before Problem 3.7, ask:

- What changes about a figure that is translated, reflected, or rotated? (its position)
- What changes about a figure that is dilated? (its size, and maybe its position)
- Do any of those transformations change the shape of the figure? (no)

During Problem 3.7 A, ask: Can you tell whether Paige rotated the figure clockwise or counterclockwise? Explain. (No, she could have rotated it either 90° clockwise or 270° counterclockwise, and the image would look the same.)

During Problem 3.7 C, ask: What does the double-prime notation of ΔA′B′C′ suggest about that triangle? (That it is a transformation of ΔA′B′C′.)

After Problem 3.7, ask: Suppose a square is rotated 90° about a point of rotation that is outside the figure. What other transformation could have been done to the square to give the same image? (translation)
**Assessment Guide for Investigation 3**

Problem 3.1, Exercises 1–3, 13  
Problem 3.2, Exercises 4–8, 13  
Problem 3.3, Exercise 12  
Problem 3.4, Exercises 9–11, 13  
Problem 3.5, Exercises 14, 16–17  
Problem 3.6, Exercises 15, 18  
Problem 3.7, Exercises 19–25

**Answers to Investigation 3**

**Problem 3.1**

A. 

B. 1. The lengths are the same.

   2. The slopes are the same.

   3. Each vertex of the triangle has been moved the same distance and in the same direction, so the entire triangle also has moved the same distance and direction.

C. 1. The angle measures are equal.

   2. Translations do not change the measures of angles in figures.

D. Every point on the line would move down 2 units and to the left 3 units. The translated line would be the line through the points $A'$ and $C'$.

**Problem 3.2**

A. 1. 2 units

   2. Point $G'$ also is 2 units from the line of reflection.

   3. Yes

   4. To reflect a point across a line, plot a point on the opposite side of the line that is the same distance from the line as the original point.

B. 1. The reflection will be a line that passes through points $G'$ and $H'$.

   2. To reflect a line across a line, select two points on the line and plot their reflections on the opposite side of the line of reflection. Then draw a line passing through those two reflected points.

C. 1. One triangle fits on top of the other.

   2. The two distances are the same.

   3. The distances for the other pairs of points also are the same.

   4. The lengths of the reflected sides are the same as the corresponding lengths of the original figure.
5. The angle measures of the reflected figure are the same as the corresponding angle measures of the original figure.

6. To reflect a polygon across a line, reflect each vertex of the figure across the line, and then connect the vertices to make the reflected polygon.

**Problem 3.3**

A. 1. The lengths are the same.
2. $KO = K'O$; $KL = K'L'$; $LM = L'M'$
3. The distance does not change.
4. The center of rotation does not change position.
5. No, each direction of a $180^\circ$ rotation results in the same transformed figure.

B. Rotate each vertex of the polygon and then connect the rotated vertices to form the rotated polygon.

C. $(0, -6)$

D. Sample: Point $L$ begins the rotation 2 units behind $(0, 6)$, so it also will end the rotation 2 units behind, or to the right of, $(0, -6)$.

E. 1. The angle measures in each pair are equal.
2. Rotations do not change the measures of angles.

F. The line would rotate so that it passes through points $L'$ and $M'$.

**Problem 3.4**

A. The distances are the same.
B. No
C. $90^\circ$

**Problem 3.5**

A. enlargement
B. 1. 2 times
2. 2 times
3. 2
4. The ratio of any linear measurement in the original figure to the corresponding measurement in the dilated image.

C. It did not change position.

**Problem 3.6**

A. reduction
B. 1. $1 : 4$
2. $1 : 4$
3. Draw a line from the center of dilation through a vertex on the polygon. Draw a line segment in that line with one endpoint at the center of dilation and with a length that corresponds to the scale factor. Repeat for the remaining vertices of the original polygon. Connect the dilated vertices to form the dilated polygon.

**Problem 3.7**

A. 1. $90^\circ$ clockwise
2. $A'(1, 5), B'(7, 5), C'(1, 1)$
3. 
4. 

B. 1. $2 : 1$
2. $2 : 1$
3. $2 : 1$

4. $\frac{1}{2}$; point $C'$
5. C. The triangles are the same, so $\triangle ABC$ and $\triangle DEF$ are similar.

Exercises
1. a. [Diagram of triangles]

2. a. 5 units to the right and 4 units up
   b. Sample: (2, 1); (7, 5)
   c. No, every point on the sides between the vertices, an infinite number, also were translated when the sides were translated.

3. a. The rule states that every point on the figure should move 1 unit to the right and 4 units down.

4. [Diagram of triangles]

5. a. C
   b. $y = 5$

6. a. the middle pair
   b. Yes
   c. Check students’ work.
7. Sample:

8. Leah’s reflection will be more difficult to draw because Ron’s line of reflection is along one side of the rectangle, but Leah’s line of reflection is at an angle to her triangle.

9. Figure $A_2B_2C_2D_2$, since the distances from the origin to the pairs of corresponding vertices are the same.

10.

11.

12. a. It can be the same if the figure is symmetric about a line that is perpendicular to the line of reflection.

   b. when the figure is not symmetric about a line that is perpendicular to the line of reflection

13. a.

   b.

   c.

   d. Yes; translated, reflected, or rotated parallel lines also are parallel.

14. enlargement; $(0, 0)$; scale factor $= 2$

15. reduction; $(-1, -2)$; scale factor $= \frac{2}{3}$

16. enlargement; $(0, 0)$; scale factor $= 1.5$
17. a.

18. a.

19. reflection, reduction

20. translation, enlargement

21. reflection, enlargement

22. reflection, translation, reduction

23. translation, reflection, or rotation, enlargement

24. translation

25. a. $A'(6, -6), B'(3, -6), C'(3, 0), D'(6, 0)$

b. Yes, the image was made through only a rotation and a dilation, so the shape of the rectangle did not change.
Additional Practice

1. Copy $\triangle ABO$ and $\triangle FGO$ onto graph paper.
   a. Draw a translation of $\triangle ABO$ 3 units left and 4 units down. Give the coordinates of $\triangle A'B'O'$.
   
   b. Draw a reflection of $\triangle ABO$ across the y-axis. Give the coordinates of $\triangle A'B'O'$.
   
   c. Draw a reflection of $\triangle FGO$ across the line $x = 1$. Give the coordinates of $\triangle F'G'O'$.
   
   d. Draw a rotation of $\triangle ABO$ 90° clockwise around the origin. Give the coordinates of $\triangle A'B'O'$.
   
   e. Draw a rotation of $\triangle FGO$ 180° around a point located at $(1, 1)$. Give the coordinates of $\triangle F'G'O'$.
   
   f. Draw a dilation of $\triangle ABO$ with a scale factor of $\frac{1}{2}$ and a center of dilation at the origin. Give the coordinates of $\triangle A'B'O'$.
   
   g. Draw a dilation of $\triangle FGO$ with a scale factor of 2 and a center of dilation located at $(1, -2)$. Give the coordinates of $\triangle F'G'O'$.

2. a. Describe the transformations that could be made to $\triangle ABO$ to result in $\triangle FGO$.
   
   b. Give the coordinates of $\triangle A'B'O'$, which is the image of $\triangle ABO$ after the first transformation you describe in Part (a).
   
   c. What does the fact that you can transform $\triangle ABO$ to become $\triangle FGO$ tell you about how $\triangle ABO$ and $\triangle FGO$ are related?
Skill: Identify Transformations

Identify the transformation shown by each figure.

1. 2. 3. 4. 5. 6.

Skill: Transformations on the Coordinate Plane

Give the coordinates of points $A'$, $B'$, and $C'$ after each transformation of $\triangle ABC$.

7. translation 3 units up
8. translation 2 units left and 2 units down
9. reflection across the $y$-axis
10. reflection across the line $x = -1$
11. rotation $90^\circ$ clockwise around the origin
12. rotation $90^\circ$ counterclockwise around point $C$
13. dilation with a scale factor of 2 and a center of dilation at the origin.
14. dilation with a scale factor of $\frac{1}{2}$ and a center of dilation at point $A$
Check-Up

Copy parallel lines $\overline{PC}$ and $\overline{RW}$ onto graph paper.

1. a. Draw translations of $\overline{PC}$ and $\overline{RW}$ 4 units down. Give the coordinates of points $P'$, $C'$, $R'$, and $W'$.

b. Compare the image of a translation of $\overline{PC}$ 3 units to the right to the original figure.

c. Draw reflections of $\overline{PC}$ and $\overline{RW}$ over the $x$-axis. Give the coordinates of points $P'$, $C'$, $R'$, and $W'$.

d. Draw rotations of $\overline{PC}$ and $\overline{RW}$ 90° clockwise around point $C$. Give the coordinates of points $P'$, $C'$, $R'$, and $W'$.

e. Compare the images you drew in Part (d) to the images of a 90° counterclockwise rotation around point $C$. Explain how they are alike and different.

f. Describe the relationship between $\overline{P'C'}$ and $\overline{RW'}$ after each transformation in Parts (a), (c), and (d).

2. Look at each of the transformations of $\overline{PC}$ in Part 1. For each, describe how $\overline{PC}$ was transformed to become $\overline{P'C'}$. 

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Copy figure $ABCD$ onto graph paper.

3. **a.** Draw a rotation of the figure 180° about (4, 3). Give the coordinates of vertices $A'$, $B'$, $C'$, and $D'$.

   b. Draw a rotation of the figure 90° counterclockwise about the origin. Give the coordinates of vertices $A'$, $B'$, $C'$, and $D'$.

   c. Draw a reflection of the figure across the line $x = 1$. Give the coordinates of vertices $A'$, $B'$, $C'$, and $D'$.

   d. Draw a dilation of the figure with a scale factor of $\frac{1}{2}$ and a center of dilation at the origin. Give the coordinates of vertices $A'$, $B'$, $C'$, and $D'$.

   e. Give the coordinates of vertices $A'$, $B'$, $C'$, and $D'$ after a dilation of figure $ABCD$ with a scale factor of 2 and a center of dilation of $(2, 1)$.

4. **a.** Draw a rotation of the figure 90° clockwise about (2, 1). Label the image $A'B'C'D'$. Then translate $A'B'C'D'$ 4 units left and 3 units up. Give the coordinates of vertices $A''$, $B''$, $C''$, and $D''$.

   b. Draw a translation of the figure 5 units left and 1 unit down. Label the image $A'B'C'D'$. Then dilate $A'B'C'D'$ with a scale factor of $\frac{1}{2}$ and a center of dilation at the origin. Give the coordinates of vertices $A''$, $B''$, $C''$, and $D''$.

   c. Describe the relationships between figures $ABCD$ and $A''B''C''D''$ in Parts (a) and (b). Explain how you know.
A transformation is the change in the size or position of a figure. A translation is a transformation in which each point of a figure moves the same distance and in the same direction. This design contains many such figures.

Problem 3.1

A. Copy \( \triangle ABC \) and translate it to \( \triangle A'B'C' \) using the steps below.

1. From \( A \), count down 2 units and to the left 3 units. Label the new point \( A' \) (A-prime).
2. Find and label points \( B' \) and \( C' \) by counting down 2 units and left 3 units.
3. Draw \( \triangle A'B'C' \).

B. Draw line segments from \( A \) to \( A' \), from \( B \) to \( B' \), and from \( C \) to \( C' \).

1. Compare the lengths of the three line segments.
2. Compare the slopes of the three line segments.
3. Explain why \( \triangle A'B'C' \) is a translation of \( \triangle ABC \).

C. Look at the angles of \( \triangle ABC \) and \( \triangle A'B'C' \).

1. Compare \( \angle A \) to \( \angle A' \), \( \angle B \) to \( \angle B' \), and \( \angle C \) to \( \angle C' \). What do you notice about each angle pair?
2. What does this tell you about the effect that translations have on angles?

D. Draw a line through points \( A \) and \( C \). Explain how that line would be translated using the same translation as for \( \triangle ABC \).
A reflection is a transformation that flips an image over a line called the line of reflection. Images shown in mirrors are reflections.

**Problem 3.2**

A. The reflection of two points across the line $y = 3$ is shown. Point $G'$ is the reflection of point $G$. Point $H'$ is the reflection of point $H$.

1. What is the shortest distance from $G$ to the line of reflection?
2. Compare your answer to the distance from $G'$ to the line of reflection.
3. Does the same comparison hold true for $H$ and $H'$?
4. Write a rule for reflecting a point across a line.

B. Suppose a line is drawn through points $G$ and $H$.

1. Describe the reflection of $GH$ across the line $y = 3$.
2. Write a rule for reflecting a line across another line.

C. The reflection of a triangle across the line $x = 4$ is shown below.

1. Imagine folding the graph over the line $x = 4$. What would happen?
2. Compare the distance from vertex $B$ to the line of reflection with the distance from vertex $B'$ to the same line.
3. Compare the distances of $C$ and $C'$ to the line of reflection. Do the same for $D$ and $D'$.
4. What do you notice about the lengths of the corresponding sides of the triangles?
5. What do you notice about the corresponding angles of the triangles?
6. Write a rule for the reflection of a polygon across a line.
A rotation is a transformation that turns a figure around a fixed point called the **center of rotation**. A rotation is **clockwise** if its direction is the same as that of a clock hand. A rotation in the other direction is called **counterclockwise**. A complete rotation is 360°.

A Ferris wheel makes a 90° rotation with every \( \frac{1}{4} \) turn.

The rotation of figure **KLMO** 180° about (0, 0) is shown. In **K'L'M'O**, point **K'** is the rotation of point **K**, point **L'** is the rotation of point **L**, and point **M'** is the rotation of point **M**.

A. 1. Compare the lengths of \( \overline{OM} \) and \( \overline{OM'} \).
   
   2. What other pairs of side lengths can you find that have the same lengths?
   
   3. When a point is rotated, how does its distance from the center of rotation change?
   
   4. Describe the movement of the point at the center of rotation.
   
   5. When you rotate a figure 180°, does it matter whether you rotate clockwise or counterclockwise?

B. Describe how to rotate a polygon by using its vertices.

C. What is the new location of the point at (0, 6) after it has been rotated clockwise 180° about the origin?

D. If you think of (0, 6) and point **L** rotating together, how can that help you understand the position of **L'**?

E. 1. Compare \( \angle K \) to \( \angle K' \), \( \angle L \) to \( \angle L' \), and \( \angle M \) to \( \angle M' \). What do you notice about each angle pair?

   2. What effect do rotations have on angles?

F. Explain how a line through points **L** and **M** would be transformed using the same transformation as for figure **KLMO**.

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**Notes**

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The clockwise rotation of \(\triangle ABC\) 90° about the origin is shown at the right.

**A.** Compare the distances from the origin to points \(C\) and \(C'\).

**B.** When a figure is rotated, does a vertex have to be the center of rotation?

**C.** If you draw a line from \(A\) and a line from \(A'\) through the center of rotation, what is the measure of the angle formed at the intersection of the lines?

A **dilation** is a transformation of a figure that changes its size but not its shape. The **scale factor** of a dilation determines the extent of the change in size. A dilation is an enlargement when the scale factor is greater than 1. It is a reduction when the scale factor is less than 1. When you dilate a figure, you shrink or enlarge a figure from the **center of dilation**.

The graph shows a dilation of \(\triangle AOB\) to \(\triangle A'OB'\) with the center of dilation at the origin.

**A.** Is \(\triangle A'OB'\) an **enlargement** or a **reduction** of \(\triangle AOB\)?

**B.** 1. How many times greater is \(OA'\) than \(OA\)?
   2. How many times greater is \(OB'\) than \(OB\)?
   3. How many times greater is \(A'B'\) than \(AB\)?
   4. The scale factor in a dilation measures the comparative size of linear measures in a figure before and after dilation. What is the scale factor of this dilation?
   5. When you are examining a dilation, what is the least information you need in order to determine the scale factor?

**C.** What happened at the center of dilation in this transformation of \(\triangle AOB\)?

**D.** Draw \(\triangle LOM\) with vertices \(L(0, 4), O(0, 0),\) and \(M(2, 0)\). Then draw \(\triangle L'OM'\) as a dilation of \(\triangle LOM\) with the center of dilation at \((0, 0)\) and a scale factor of 1.5.
The graph shows the dilation of figure $KLMN$ to $K'L'M'N'$ with the center of dilation at $C(2, 2)$ and a scale factor of $\frac{3}{4}$.

A. Is $K'L'M'N'$ an enlargement or a reduction of $KLMN$?

B. 1. What is the ratio of $K'L'$ to $KL$?
2. What is the ratio of the length of $CN'$ to the length of $CN$?
3. Describe a strategy for drawing the dilation of a polygon when you know the scale factor. (Hint: $CN$ and $CN'$ lie on the same line.)

C. Make a copy of $KLMN$ and draw a reduction with the center of dilation at $(2, 2)$ and a scale factor of $\frac{3}{4}$.

Two figures are similar if they have the same shape, but not necessarily the same size. Corresponding angles of similar figures are equal, and corresponding sides are proportional. Transformations can change the orientation and size of a figure, but they do not change its shape. A transformed figure is always similar to the original figure.

Paige says she can prove that $\triangle ABC$ and $\triangle DEF$ are similar by making only two transformations of $\triangle ABC$.

A. First, Paige rotates $\triangle ABC$ about the origin.
1. How much of a rotation, and in which direction, should Paige rotate $\triangle ABC$ so that the image of point $C$ is at point $F$?
2. Give the coordinates of points $A'$, $B'$, and $C'$ after the rotation.
3. Draw $\triangle A'B'C'$ after the rotation.
B. Next, Paige dilates \( \triangle A'B'C' \).
   1. Write a ratio to compare the lengths of \( AB \) and \( DE \).
   2. Write a ratio to compare the lengths of \( CA' \) and \( F'D \).
   3. What scale factor should Paige use? Where will the center of dilation be?
   4. Draw the dilation of \( \triangle A'B'C' \). Label the dilation \( \triangle A''B''C'' \).

C. Compare \( \triangle A''B''C'' \) and \( \triangle DEF \). Explain what the comparison tells you about \( \triangle ABC \) and \( \triangle DEF \).

Exercises

1. For each of the directions below, copy the graph and translate \( \triangle RST \). Label the image \( \triangle R'S'T' \).
   a. Translate \( \triangle RST \) up 2 units.
   b. Translate \( \triangle RST \) to the right 2 units.
   c. Translate \( \triangle RST \) to the left 2 units and down 4 units.
   d. Translate \( \triangle RST \) to the right 1 unit and down 1 unit.
   e. Translate \( \triangle RST \) to the left 2 units and up 1 unit.

2. Danielle drew the figures at the right to represent a translation.
   a. Describe the translation of point \( E \) to point \( E' \).
   b. Name the coordinates of an unlabeled point on the bottom figure, then give the coordinates of the translated image of that point.
   c. Jeremy says that Danielle only plotted 6 points to do the translation, so that means only 6 points on the original figure were translated. Do you agree with Jeremy?

3. Chee wrote the rule \( (x, y) \rightarrow (x + 1, y - 4) \) to describe the translation of \( \triangle ABC \) to \( \triangle A'B'C' \).
   a. Describe how each point on \( \triangle ABC \) will move, using Chee’s rule.
   b. Chee drew \( \triangle KLM \) with vertices at \( K(2, 7), L(4, 6), \) and \( M(3, 4) \). He then followed his own rule to draw \( \triangle K'L'M' \). Draw the triangles.
4. Copy $\triangle MNP$ on graph paper and graph its image after a reflection across the line $x = 3$.

5. Evie drew three transformations of figure $A$.

   a. Which figure is the reflection?
   b. What is the line of reflection?

6. Two of the pairs of letters show a reflection.

   a. Which pair does not show a reflection?
   b. Can any letter be flipped over a line of reflection?
   c. Flip your printed name over a line of reflection.

7. Tiara reflected the figure at the right and Deena translated it. Their new figures ended up in exactly the same location. Draw Tiara’s reflected figure.

8. Ron and Leah wanted to show a reflection over a line by tracing a flat shape then flipping it over the line and tracing it again. Whose reflection will be more difficult to draw?

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**Notes**

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9. In the diagram below, which figure is a rotation of figure $ABCD$? Explain how you know.

10. Copy $\triangle EFG$ onto graph paper and draw $\triangle E'F'G'$ as its image after a clockwise rotation of $180^\circ$ about the origin.

11. Copy the figure below onto graph paper and draw its image after a counterclockwise rotation of $90^\circ$ about the origin.

12. a. Explain how a $180^\circ$ rotation of a shape can have the same end result as a reflection.
   b. How can you tell when a $180^\circ$ rotation will give a different end result?
13. Copy parallel lines \(a\) and \(b\) onto graph paper.

\[
\begin{array}{c|c|c|c|c|c|c}
\hline
& -4 & -2 & 2 & 4 & 6 \\
\hline
\hline
\hline
a & & & & & \\
\hline
\hline
b & & & & & \\
\hline
\hline
\end{array}
\]

a. Draw a translation of the lines 3 units down.
b. Draw a reflection of the lines across the \(x\)-axis.
c. Draw a rotation of the lines \(90^\circ\) clockwise about the origin.
d. Were lines \(a'\) and \(b'\) parallel after each transformation? What does this tell you about translations, reflections, and rotations of parallel lines?

For Exercises 14–16, identify each as an enlargement or reduction. Name the coordinates of the center of dilation and give the scale factor.

14. 

15. 

16. 

17. For a dilation centered at the origin you can find the location of points on the dilated image by multiplying the coordinates on the original image by the scale factor. Use this technique to draw the dilation of the quadrilateral. Use a scale factor of \(\frac{3}{2}\) and a center of dilation at \((0, 0)\).

18. a. Draw \(\triangle ABC\) with vertices at \((-5, -1)\), \((1, 3)\), and \((1, -1)\).
b. Dilate \(\triangle ABC\) with a scale factor of \(\frac{1}{2}\) and a center of dilation at \((1, 3)\).
For Exercises 19–24, describe a sequence of transformations that proves figure A is similar to figure B.

19. 

20. 

21. 

22. 

23. 

24. 

25. Rotate \(ABCD\) 180° about the origin, then dilate it by a factor of 3 with the center of dilation at \((0, 0)\).

a. What are the coordinates of \(A', B', C',\) and \(D'\)?

b. Are the rectangles similar? Explain how you know.
CC Investigation 4: Geometry Topics

Mathematical Goals

- Use informal arguments to establish facts about the angle sum and exterior angle of triangles.
- Use informal arguments to establish facts about the angles created when parallel lines are cut by a transversal.
- Use informal arguments to establish facts about the angle-angle criterion for similarity of triangles.
- Know the formulas for the volumes of cones, cylinders, and spheres, and use them to solve problems.

Teaching Notes

This investigation reviews concepts students studied in the Grade 8 Units *Shapes and Designs* and *Filling and Wrapping*.

In this investigation, students find relationships between the angles formed when parallel lines are cut by a transversal, and then apply those relationships to find unknown angle measures. Before students begin the topic, review the definitions of acute, right, obtuse, and straight angles, and the fact that congruent angles have the same measure. Also make sure students know how to correctly use the angle ruler or protractor to measure angles.

Begin by asking students to name some real-world examples of parallel lines (train tracks, parallel bars) and of parallel lines cut by a transversal (parallel streets intersected by a third street). Show an example of the latter. Without calling attention to any of the angles, ask students to describe any apparent symmetries or patterns they see. Have them discuss methods they could use to find out whether their observations are true in general for parallel lines cut by a transversal or whether they are limited to the example you have shown.

Problem 4.1

As an alternative to using lined paper in Problem 4.1 A, students can draw a pair of parallel lines on a blank sheet of paper, using the long sides of an index card or a business envelope as guides.

Vocabulary
- supplementary angles
- alternate angles
- interior angles
- exterior angles
- corresponding angles
- vertical angles
- volume

Materials
- lined paper
- pencil
- straightedge
- angle ruler or protractor
- blank paper (optional)
- index card or business envelope (optional)
- Labsheet Exercise 33
As students work on Problem 4.1 A, guide them to think about how the angles are changing as they move the transversal. Make sure students focus on whether angle measures are increasing or decreasing, and how the measures of other angles are changing at the same time, rather than what the exact angle measures are.

During Problem 4.1 A, ask:

- As the measure of ∠1 increases when you move the transversal, do any other angle measures increase? (yes, the measures of ∠4, ∠5, and ∠8)
- Why do the measures of those angles increase along with the measure of ∠1? (They are located in positions similar to that of ∠1 relative to the transversal and the parallel lines.)

After Problem 4.1 B, ask: When you measured the angles, did you find any sets of congruent angles? If so, which angles were they? (Yes; ∠1, ∠4, ∠5, and ∠8 are congruent, and ∠2, ∠3, ∠6, and ∠7 are congruent.)

Before Problem 4.1 C, Part 1, have students read the bulleted terms. Ask students if they are familiar with the everyday meaning of each term. Ask: What does the word interior mean? the word exterior? (something that is inside; outside) Then ask:

- For the angles, what do you think exterior and interior mean in relation to the parallel lines? (interior: between the lines; exterior: outside the lines)
- What can you look for to find corresponding angles? alternate angles? (angles that are in similar positions; angle pairs where the angles are on opposite sides of a line)
- A supplement is something that is added to make something else complete. Look at angles 2, 3, and 4. Two of these angles are supplementary to angle 1. Which two? Why? (∠2 and ∠3; added to ∠1, each makes a straight angle.)
- The third angle forms a vertical angle with angle 1. Which one? (∠4)

During Problem 4.1 C, Part 2, ask: If angle 2 measures 65°, are there any other angles that also measure 65°? (yes; ∠3, ∠6, and ∠7)

Problem 4.2

Before Problem 4.2 A, have students study the figure. Ask: What do you know about parallelograms? (Opposite sides are parallel, opposite angles are congruent.)

During Problem 4.2 A, guide students to see that Parallel Way is a transversal that intersects two parallel sides of the rose garden. Ask: What can help you find the measure of ∠c or ∠d? (∠c and the 118° angle are corresponding angles; ∠d and the 118° angle are supplementary angles.)

As students work on Problem 4.2 C, ask: What do you notice about the stone walls? (They form transversals to the parallel boundaries of the park.) Have students list congruent angle pairs. Then ask: How can you find the measure of ∠j? of ∠e? (m∠j = 180° - (118° + 28°); m∠e = 118° - 76°)
Problem 4.3
Before Problem 4.3, have students study the figure. Ask: What do you notice about each of the straight rows of bushes? (They are transversals that cut the parallel paths, New Path and East Path.) Help students see the two triangles formed by the rows of bushes and paths.

Before Problem 4.3 B, ask: What do you know about similar triangles? (Corresponding angles have equal measures.)

During Problem 4.3 B Part 2, ask: What is the sum of the measures of all angles of a triangle? (180°).

Problem 4.4
Before Problem 4.4, during Getting Ready, ask: How do you define a cylinder and a cone? (A cylinder is a three-dimensional shape with two parallel and congruent circles as bases; a cone has one base that is a circle and one vertex.) Guide students to understand that using the value 3.14 for π gives an approximation of the volume. Ask: How is a volume that is found using 3.14 for π different than a volume that is given using the term π? (Using 3.14 gives an estimation of the volume, while using the term π gives the exact volume.)

During Problem 4.4 A, ask: What three-dimensional shape will a circular pool have? (cylinder)

During Problem 4.4 B, Part 1, ask: How can you set the volumes of the two pieces of the sculpture equal to each other to write an equation that you can use to solve this problem? (The volume of the sphere is \(\frac{4}{3}\pi r^3\), and the volume of the cone is \(\frac{1}{3}\pi r^2h\). The radius of the sphere is one-half the height of the cone, or 15 cm. Write the equation \(\frac{4}{3}\pi (15)^3 = \frac{1}{3}\pi r^2(30)\), and solve for \(r\).)

Summarize
To summarize the lesson, ask:

- What do you know about the angles that are formed when parallel lines are cut by a transversal? (Four congruent angles are formed, and four congruent angles supplementary to the first set of angles also are formed.)

- How is the measure of an exterior angle of a triangle related to the triangle’s interior angles? (The measure of an exterior angle is equal to the sum of the opposite angles of the triangle.)

- The measure of how many corresponding angles of two triangles need to be known to determine that the triangles are similar? (2 out of the 3)
Assignment Guide for Investigation 4

Problem 4.1, Exercises 1–8, 17
Problem 4.2, Exercises 9–10
Problem 4.3, Exercises 11–16, 18–34
Problem 4.4, Exercises 35–41

Answers to Investigation 4

Problem 4.1

A. 1. a. The measure of \( \angle 1 \) increases and the measure of \( \angle 2 \) decreases as you move the pencil.
   b. \( \angle 5 \) and \( \angle 6 \)

2. a. The measures of \( \angle 1 \) and \( \angle 4 \) remain equal as the pencil moves.
   b. \( \angle 5 \) and \( \angle 8 \), \( \angle 2 \) and \( \angle 3 \), or \( \angle 6 \) and \( \angle 7 \)

3. They have the same measure and maintain the same measure as the pencil moves.

4. They have the same measure and maintain the same measure as the pencil moves.

B. 1. Check students’ drawings and measurements. Angles 1, 4, 5, and 8 should have the same measure. Angles 2, 3, 6, and 7 should have the same measure.

2. Sample answer: Yes; the measures of \( \angle 1 \), \( \angle 4 \), \( \angle 5 \), and \( \angle 8 \) are the same, so these angles are congruent. The measures of \( \angle 2 \), \( \angle 3 \), \( \angle 6 \), and \( \angle 7 \) are the same, so these angles are also congruent.

C. 1. a. Group 1: exterior supplementary angles;
   Group 2: alternate interior angles;
   Group 3: alternate exterior angles;
   Group 4: corresponding angles;
   Group 5: supplementary angles;
   Group 6: vertical angles
   b. Group 4: \( \angle 1 \) and \( \angle 5 \), \( \angle 4 \) and \( \angle 8 \);
   Group 5: \( \angle 1 \) and \( \angle 2 \), \( \angle 2 \) and \( \angle 4 \), \( \angle 7 \) and \( \angle 8 \), \( \angle 5 \) and \( \angle 6 \), \( \angle 5 \) and \( \angle 7 \);
   Group 6: \( \angle 5 \) and \( \angle 8 \), \( \angle 6 \) and \( \angle 7 \)

2. a. \( \angle 3 \), \( \angle 6 \), and \( \angle 7 \) are congruent to \( \angle 2 \), so they all measure 65\(^\circ\); \( \angle 1 \) and \( \angle 2 \) are supplementary, so \( m\angle 1 = 180^\circ - 65^\circ = 115^\circ \); \( \angle 4 \), \( \angle 5 \), and \( \angle 8 \) are all congruent to \( \angle 1 \), so they all measure 115\(^\circ\).
   b. Check students’ measurements.

Problem 4.2

A. Parallel Way and New Path are both transversals that intersect two parallel sides of the rose garden, so Marsha can use the relationships between angles formed by parallel lines and transversals; or Marsha can use the fact that \( \angle d \) and the 118\(^\circ\) angle are supplementary, so \( m\angle d = 180^\circ - 118^\circ \). Then she can use the fact that parallelograms have two pairs of opposite congruent angles and that the sum of the angle measures is 360\(^\circ\) to find the other angle measures.

B. \( m\angle d = m\angle b = 62^\circ \); \( m\angle a = m\angle c = 118^\circ \).

C. \( m\angle e = 118^\circ - 76^\circ = 42^\circ \); \( m\angle j = 62^\circ - 28^\circ = 34^\circ \); \( \angle f \) is an alternate interior angle with the given angle that measures 28\(^\circ\) so \( m\angle f = 28^\circ \), which means \( m\angle g = 62^\circ - 28^\circ = 34^\circ \); \( \angle h \) is an alternate interior angle with the given angle that measures 76\(^\circ \) so \( m\angle h = 76^\circ \); \( m\angle i = 118^\circ - 76^\circ = 42^\circ \)
Problem 4.3
A. 1. They are vertical angles.
2. \( m \angle y = 47^\circ \)
3. They are alternate interior angles.
4. \( m \angle x = 72^\circ \)
5. They are alternate interior angles.
B. 1. Yes, the corresponding angles of the triangles are: \( y \) and the angle measuring \( 47^\circ \); \( x \) and the angle measuring \( 72^\circ \); \( w \) and \( z \), which are alternate interior angles that have the same measure.
2. The angle sum of a triangle is \( 180^\circ \);
3. The measures of alternate interior angles are equal, so \( m \angle z = m \angle w = 61^\circ \).
C. Yes; \( x \) and \( a \) are corresponding angles that have equal measures, and \( w \) and \( b \) also are corresponding angles having equal measure. Two corresponding pairs of angles are equal, so the triangles are similar.

Problem 4.4
A. 1. \( V = \pi r^2 h = \pi (5^2)(3) = 75\pi \text{ ft}^3 \); \( V = 75(3.14) = 236 \text{ ft}^3 \)
2. \( V = \pi r^2 h; 750 = (3.14)r^2(2); r = 10.9 \text{ ft} \)
3. To have the largest radius and look largest in the garden, the pool needs to have the least depth, 1 ft. \( V = \pi r^2 h; 750 = (3.14)r^2(1); r = 15.5 \text{ ft} \). The pool should be 1 ft deep and have a radius of about 15.5 ft.
B. 1. \( r = 21.2 \text{ cm} \)
2. The height of the cone should be 4 times its radius.

Exercises
1. \( \angle 3 \) and \( \angle 6, \angle 4 \) and \( \angle 5 \)
2. \( \angle 1 \) and \( \angle 4, \angle 2 \) and \( \angle 3, \angle 5 \) and \( \angle 8, \angle 6 \) and \( \angle 7 \)
3. \( \angle 1 \) and \( \angle 5, \angle 4 \) and \( \angle 8, \angle 3 \) and \( \angle 7, \angle 2 \) and \( \angle 6 \)
4. \( \angle 2, \angle 3 \), \( \angle 6 \), and \( \angle 7 \)
5. \( \angle 4, \angle 5, \) and \( \angle 8 \) measure \( 80^\circ \); \( \angle 2, \angle 3, \angle 6 \), and \( \angle 7 \) measure \( 100^\circ \).
6. C
7. The marked angle measures \( 90^\circ \). The corresponding angle at the intersection of Smith and Acorn must also have measure \( 90^\circ \), so they are perpendicular.
8. \( x = 60 \)
9. \( \overline{AD} \), \( \overline{BC} \), and \( \overline{AC} \)
10. \( m \angle 3 = 28^\circ \)
11. \( x = 73 \)
12. \( k = 140^\circ, m = 110^\circ, n = 70^\circ, p = 40^\circ \)
13. \( x = 25^\circ, y = 75^\circ \)
14. \( a = 95^\circ \)
15. \( n = 20; \) each angle measures \( 110^\circ \).
16. \( a = 120^\circ \)
17. They are congruent.
18. Yes, the triangles are similar.
19. No, the triangles are not similar.
20. Yes, the triangles are similar.
21. Yes, the triangles are similar.
22. No; the missing angle measure of the first triangle is \( 180^\circ - 90^\circ - 38^\circ = 52^\circ \), so the triangles have at least two corresponding angles with measures that are not equal.
23. Yes; \( \overline{AC} \) is a transversal to parallel segments \( \overline{AB} \) and \( \overline{DE} \), so corresponding angles \( A \) and \( D \) have the same measure; \( \overline{DF} \) is a transversal to parallel segments \( \overline{BC} \) and \( \overline{EF} \), so corresponding angles \( C \) and \( F \) have the same measure.
24. \( 76^\circ, 27^\circ \)
25. No; the unknown angle measure for the second triangle is \( 180^\circ - 61^\circ - 58^\circ = 61^\circ \).
26. $67^\circ$
27. $72^\circ$
28. $52^\circ$
29. $72^\circ$
30. $55^\circ$
31. $120^\circ$
32. $18^\circ$

33. a. $180^\circ$; a straight angle measures $180^\circ$.
   b. They are alternate interior angles. Their measures are equal; $m\angle 1 = 30^\circ; m\angle 4 = 30^\circ$
   c. They are alternate interior angles. Their measures are equal; $m\angle 3 = 60^\circ; m\angle 5 = 60^\circ$
   d. $90^\circ$
   e. They are the angles of a triangle; the sum of their measures is $30^\circ + 60^\circ + 90^\circ = 180^\circ$.
   f. $180^\circ$

34. a. They are the angles of a triangle; the sum of their measures is $180^\circ$.
   b. $180^\circ$; a straight angle measures $180^\circ$.
   c. The left side of the equation, $w + x + y$, represents the sum of the angles of a triangle, which is $180^\circ$; the right side of the equation, $x + z$, represents the sum of the angles that make a straight angle, which also is $180^\circ$.
   d. After subtracting, the equation is $w + y = z$. The left side of the equation, $w + y$, represents the sum of the measures of the opposite interior angles to $\angle x$; the right side of the equation is $z$, which is the measure of the exterior angle to $\angle x$.

35. $\frac{256\pi}{3}$ in.$^3$
36. $135\pi$ ft$^3$
37. $768\pi$ cm$^3$
38. $36\pi$ cm$^3$
39. 24 cones
40. a. about 11.5 in.$^3$
   b. about 40 times
41. $\frac{250\pi}{3}$ in.$^3$
1. The local airport is putting in a new terminal and new runways. Part of a diagram of the new runways is shown below. Runways 2W and 4W are parallel.

a. One surveyor measures $\angle c$ to be 40°. Another measures $\angle f$ to be 144°. Explain how you know, without measuring any of the angles, that at least one of the surveyors is incorrect.

b. What has to be true about the relationship between runway 18N and runways 2W and 4W in order for $\angle k$ and $\angle n$ to have the same measure? Explain using the relationships of angles around transversals.

c. Describe the relationship between the triangle that includes $\angle a$ and $\angle c$, and the triangle that includes $\angle a$ and $\angle g$. Explain how you know.

d. If $m\angle c = 40^\circ$, then what is $m\angle g$? Explain how you know.

2. Contestants race to fill a 1-gallon bucket with water they scoop and carry from a lake. Each contestant can pick from one of these cylindrical or cone-shape scoops.

a. Which scoop would be the best to use for the race? Explain your answer.

b. How much more water could a contestant carry in 10 trips from the lake with the scoop you selected in Part A than the second-best scoop? Show your work.
Skill: Measure Angles

Find the measure of the angle. Lines $a$ and $b$ are parallel.

1. $\angle s$
2. $\angle t$
3. $\angle u$
4. $\angle v$
5. $\angle w$
6. $\angle x$
7. $\angle y$
8. $\angle z$

Skill: Evaluate Expressions

Find the value of the expression. Give your answer to the nearest hundredth.

9. $180 - 60$
10. $90 - 45$
11. $36 + 144$
12. $6^2$
13. $(2.5)^2$
14. $\left(\frac{1}{2}\right)^2$
15. $8^3$
16. $(4.2)^3$
17. $\left(\frac{3}{4}\right)^3$
18. $\frac{1}{3} \times 18$
19. $15.9 \times \frac{1}{3}$
20. $8 \times \frac{4}{3}$
21. $\frac{4}{3} \times 19.7$
22. $4\pi$
23. $13.3\pi$
24. $140\pi$
Check-Up

1. The map shows some paths through a corn maze. Path B is parallel to Path C.
   a. List each pair of vertical angles.

   b. List each pair of corresponding angles.

   c. List each pair of alternate interior angles.

   d. List each pair of alternate exterior angles.

   e. List 4 pairs of angles that are supplementary.

   f. Milagros uses a protractor to find that the measure of angle $a$ is $140^\circ$.
      To get through the maze, she thinks it might be helpful to know the other angle measures. Without measuring them, find the measures of angles $b$ through $h$. Show your work.

2. Reece has a block of modeling clay that measures 3 in. by 3 in. by 7 in. He divides the clay in half and models a sphere with one piece and a cone with the other piece.
   a. If he wants to make the diameters of the sphere and the base of the cone to be the same, how tall should he make the cone? Show your work.

   b. If he makes the height of the cone the same as the sphere’s diameter, what is the diameter of the base of the cone? Show your work.
3. The Cape Cove City Board approved the proposal to build Beach Parkway, a new road parallel to Sands Boulevard. Beach Parkway will intersect Lagoon Way and Cove Street. The planned sketch is shown below.

a. The civil engineer in charge of the project needs to know the measures of the numbered angles. Find each of these measures and tell how you found them.

b. The owners of the Lake Park Zoo want to know what the angles of the new corners of their property will be. Explain how to find the measures of the other three angles of the Lake Park Zoo.

c. Give the measures of the three angles of the triangle formed by Cove Street, Lagoon Way, and Sands Boulevard.

4. Part of Beach Parkway will cross an inlet, and the city will need to construct cylindrical concrete pillars that are 4.5 ft in diameter and 20 feet tall. A cement truck carries 13.5 yd$^3$ of concrete in a single load. How many pillars can be made from one truckload of concrete? Show your work.
A transversal is a line that intersects two or more other lines. When parallel lines are intersected by a transversal, many angles are formed. They can be acute, obtuse, or have special relationships between pairs. Knowing about angle relationships is useful when you make designs.

The Riverside School students have been invited to propose designs for a small park on a rectangular plot of land next to the school. The plot has two parallel paths crossing it.

A. Juan, Marsha, and Cora decide to include in their design a new path to connect different parts of the park. The path will be diagonal and cross the two existing paths. They use a pencil to represent the diagonal path on a piece of lined paper to help decide its position. Move the pencil to different positions as shown below and observe the angles.

1. a. How does $\angle 1$ change in relation to $\angle 2$ as you move the pencil?
   b. Name another pair of angles with this same relationship.
2. a. When the pencil is moved, how does this affect the relationship between $\angle 1$ and $\angle 4$?
   b. Name another pair of angles with this same relationship.
3. Describe the relationship between $\angle 1$ and $\angle 8$.
4. Describe the relationship between $\angle 1$ and $\angle 5$. 

Common Core State Standards: 8.G.5; 8.G.9
B. Use a straightedge to draw a transversal on lined paper.
   1. Measure all of the angles.
   2. Do your measurements verify the angle relationships you
discovered in Part A? Explain.

C. Juan, Marsha, and Cora are working on the
design shown. The black paths on the design
are parallel.

   1. The terms below are commonly used when describing angles that
are formed when parallel lines are intersected by a transversal.
   - supplementary
   - alternate
   - interior
   - exterior
   - vertical
   - corresponding

   a. Use the terms and what you have observed about angle pairs to
label each group of angles shown in the table below. Some
groups can be described using more than one term.

<table>
<thead>
<tr>
<th>Group 1</th>
<th>Group 2</th>
<th>Group 3</th>
</tr>
</thead>
<tbody>
<tr>
<td>∠1 and ∠2</td>
<td>∠3 and ∠6</td>
<td>∠1 and ∠8</td>
</tr>
<tr>
<td>∠7 and ∠8</td>
<td>∠4 and ∠5</td>
<td>∠2 and ∠7</td>
</tr>
</tbody>
</table>

   b. List any angles that may be missing from each group.

   2. In the park design, ∠2 measures 65°.

   a. Show how Juan, Marsha, and Cora can use angle relationships
to find the measures of each of the other angles. List the angle
measures and justify your reasoning.

   b. Measure the angles to verify your findings.
A local nursery offers to donate rose bushes for a planned rose garden in the park. Marsha designs a rose garden shaped like a parallelogram. The rose garden will have boundaries along the paths.

A. An iron fence will surround the park. The measures of angles $a$, $b$, $c$, and $d$ must be included in the blueprint for the person making the fence. How can Marsha find these angle measures without using an angle ruler or protractor?

B. What are the measures of angles $a$, $b$, $c$, and $d$?

C. The final plan includes short stone walls that join opposite corners of the Rose Garden.

One of the walls forms a $76^\circ$ angle with New Path. The other wall will make a $28^\circ$ angle with Parallel Way. Show how Juan, Marsha, and Cora can find the measures of $\angle e$ through $\angle j$. 

Notes
Problem 4.3

Juan drew this plan to show two straight rows of bushes that will cross and connect two parallel paths, New Path and East Path. Two triangles are formed by the bushes and paths.

A. 1. What is the relationship between angle \( y \) and the angle with a measure of \( 47^\circ \)?
2. What is the measure of angle \( y \)?
3. What is the relationship between angle \( x \) and the angle with a measure of \( 72^\circ \)?
4. What is the measure of angle \( x \)?
5. What is the relationship between angles \( w \) and \( z \)?

B. Juan is trying to determine if the triangular sections are similar. Corresponding angles of similar triangles have equal measures.
1. Can Juan decide whether the triangles formed by the bushes and paths are similar knowing only the given angles and the measures of angles \( x \) and \( y \)? Explain why or why not.
2. Describe how to find the measure of angle \( w \) using the angle sum of triangles.
3. Describe how to find the measure of angle \( z \) using properties of transversals of parallel lines.
4. How many pairs of angles of the triangles need to be congruent to determine that the triangles are similar?

C. Juan drew the location of a sprinkler line that runs parallel to New Path. The new line forms a smaller triangle within the larger triangle. Are the triangles similar? Explain why or why not.
**Volume** is the amount of space enclosed in a solid figure. Volume is expressed in cubic units, such as cm\(^3\) or ft\(^3\). The exact volume of a cylinder, cone, or sphere includes the value \(\pi\).

**Getting Ready for Problem 4.4**

Use these formulas to find the volumes of cylinders, cones, and spheres.

- **Cylinder**
  \[ V = \pi r^2 h \]

- **Cone**
  \[ V = \frac{1}{3} \pi r^2 h \]

- **Sphere**
  \[ V = \frac{4}{3} \pi r^3 \]

**Problem 4.4**

**A.** Cora proposes adding a circular pool with uniform depth and a fountain to the rose garden. The budget for the park will allow for a fountain pump that operates best in a pool with a maximum volume of 750 ft\(^3\).

1. Find the exact volume of a circular pool with a radius of 5 ft and a depth of 3 ft. Approximate the pool’s volume by using the value 3.14 for \(\pi\).
2. What is the greatest radius the circular pool can have if its depth is at most 2 ft?
3. Cora wants the pool to be between 1 ft and 2 \(\frac{1}{2}\) ft deep. What dimensions should the pool have? Explain your choice.

**B.** Cora is designing a sculpture for the garden. The height of the cone will be the same as the diameter of the sphere.

1. Cora wants the volumes of the two pieces to be the same. If the height of the cone is 30 cm, what does its radius need to be?
2. Cora decides that the sculpture will look better if the radius of the cone matches the radius of the sphere. How tall should the cone be to keep the volumes of the two pieces the same?
**Exercises**

Use the figure below for Exercises 1–5. Lines $L_1$ and $L_2$ are parallel.

1. Name two pairs of alternate interior angles.
2. Name four pairs of vertical angles.
3. Name four pairs of corresponding angles.
4. Name four angles that are supplementary to $\angle 4$.
5. The measure of $\angle 1$ is $80^\circ$. Find the measures of the other angles.

6. **Multiple Choice** What is the reason that $\angle 5$ and $\angle 6$ do not form a pair of alternate interior angles?
   
   A. They are not alternate angles.
   
   B. They are not interior angles.
   
   C. They share the same vertex.
   
   D. They are not supplementary.
7. Rose Avenue is parallel to Acorn Avenue and perpendicular to Smith Street.

![Diagram of Rose Avenue, Acorn Avenue, and Smith Street]

Explain why Smith Street must be perpendicular to Acorn Avenue.

8. Find the value of $x$.

![Diagram with $x^\circ$ and $2x^\circ$]

Use parallelogram $ABCD$ for Exercises 9–10.

9. Name the pair of line segments for which $\angle 1$ and $\angle 4$ form a pair of alternate interior angles.

10. The measure of $\angle 2$ is $28^\circ$. Find the measures of any other angles in the figure that are possible to find.
11. The figure shows two parallel lines intersected by two transversals.

Find the value of $x$.

12. A surveyor has been hired to lay out the boundaries of a trapezoid-shaped park. The park’s designer has specified external angles of $140^\circ$ and $110^\circ$, as shown below.

Find the measures of $\angle k$, $\angle m$, $\angle n$, and $\angle p$.

13. The figure shows four of the runways at Metropolitan Airport.

Find the measures of $\angle x$ and $\angle y$. 
14. Two parallel lines are cut by a transversal.

\[ (3x - 10)^\circ \quad \quad a \quad \quad (2x + 15)^\circ \]

Find the measure of angle \(a\).

15. The figure shows the design of the flag of Jamaica.

\[ (n + 90)^\circ \quad (2n + 70)^\circ \]

Find \(n\) and the measures of the two angles.

16. In the figure, the two horizontal lines are parallel.

\[ 100^\circ \quad \quad a \quad \quad 40^\circ \]

Find the measure of \(\angle a\).

17. Angle 1 is supplementary to angle 2. Angle 2 is vertical to angle 3. Angle 3 is an alternate exterior angle to angle 4. Angle 4 is supplementary to angle 5. How is angle 1 related to angle 5?
For Exercises 18–21, determine whether \( \triangle ABC \) is similar to \( \triangle XYZ \).

18. \( m \angle A = 30^\circ; m \angle B = 90^\circ; \)
   \( m \angle X = 30^\circ; m \angle Y = 90^\circ \)

19. \( m \angle A = 65^\circ; m \angle C = 75^\circ; \)
   \( m \angle X = 65^\circ; m \angle Y = 75^\circ \)

20. \( m \angle B = 50^\circ; m \angle C = 70^\circ; \)
   \( m \angle X = 60^\circ; m \angle Z = 70^\circ \)

21. \( m \angle C = 70^\circ; m \angle A = 42^\circ; \)
   \( m \angle Y = 68^\circ; m \angle Z = 70^\circ \)

22. Are the triangles shown below similar? Explain why or why not.

23. In the triangles below, \( \overline{AB} \) is parallel to \( \overline{DE} \), and \( \overline{BC} \) is parallel to \( \overline{EF} \). Is \( \triangle ABC \) similar to \( \triangle DEF \)? Explain why or why not.

24. Maya drew similar triangles for a presentation. She wants the smaller triangle to be half the area of the larger triangle. She measures the smallest angle of the smaller triangle, and the largest angle of the larger triangle. The smaller triangle has one angle that measures 27°. The larger triangle has one angle with a measure of 77°. What are the other two angle measures of the larger triangle?

25. Are the triangles shown below similar? Explain why or why not.
For Exercises 26–29, use the diagram below. Lines \( l_a \) and \( l_b \) are parallel. Lines \( l_c \), \( l_d \), and \( l_e \) are parallel.

26. What is the value of \( w \)?

27. What is the value of \( x \)?

28. What is the value of \( y \)?

29. What is the value of \( z \)?

30. What is the value of \( k \) in the figure below?

31. The shape shown is an equilateral triangle. What is the value of \( e \)?

32. Hiro is covering a tabletop with tiles. He sets a tile that has the shape of an isosceles triangle in one square corner of the tabletop as shown. He needs to cut another tile to finish the corner. What is the value of \( p \)?
33. The main path through a park will be paved with a pattern of congruent triangular stones. The edges of the path are parallel.

![Diagram of path with labeled angles]

a. For the top edge of the path to be a straight line, what is the sum of the measures of $\angle 1$, $\angle 2$, and $\angle 3$? Explain how you know.

b. What is the relationship between $\angle 1$ and $\angle 4$? Measure $\angle 1$. What is the measure of $\angle 4$?

c. What is the relationship between $\angle 3$ and $\angle 5$? Measure $\angle 3$. What is the measure of $\angle 5$?

d. What is the measure of $\angle 2$?

e. Describe the relationship among $\angle 2$, $\angle 4$, and $\angle 5$. What is the sum of their measures?

f. What is the sum of the measures of the angles of any triangle?

34. Two congruent stones have been laid along a straight path.

![Diagram of stones with labeled angles]

a. Describe the relationship among $\angle w$, $\angle x$, and $\angle y$. What is the sum of their measures?

b. For the top edge of the path to be a straight line, what is the sum of the measures of $\angle x$ and $\angle z$? Explain how you know.

c. Explain why $w + x + y = x + z$.

d. Subtract $x$ from each side of the equation. Explain how the equation relates the measure of an exterior angle of a triangle to the sum of the measures of the opposite interior angles.
For Exercises 35–38, find the exact volume of the solid.

35.

36.

37. 38.

39. A cylindrical watering can does not fit under a faucet, so Trang is using a paper cone to fill it with water. The cone has a radius of 1 inch and a height of 4 inches. The can has a radius of 2 inches and a height of 8 inches. How many full cones of water will it take to fill the can?

40. A baseball has a diameter of 2.8 in.
   a. What is the volume of the baseball?
   b. About many times more volume does a basketball with a radius of 4.78 inches have than a baseball?

41. What is the exact volume of the largest cone that can fit into a cube with edges of 10 inches?
CC Investigation 5: Bivariate Data

Mathematical Goals

- Understand that patterns in bivariate categorical data can be seen by displaying frequencies and relative frequencies in two-way tables.
- Construct and interpret two-way tables summarizing bivariate categorical data.
- Use relative frequencies for rows or columns in a two-way table to describe possible associations between the two variables.

Teaching Notes

In this investigation, students will be examining patterns of association in bivariate categorical data. Review the definition of categorical data that begins the investigation. Encourage students to suggest other examples of categorical data. Categorical data also could include numbers, if the numbers were specified intervals such as the ages of children less than, greater than, or equal to 10. This type of categorical data will not be included in this investigation.

The data in this investigation also are bivariate. A single population is surveyed about two different topics, and then data are categorized in a two-way table. Establish the principle with students that the data must come from the same population for an association to exist.

Problem 5.1

During Problem 5.1 A, work with students to determine the number of responses in each category. Ask:

- What are the two variables Alejandra is considering? (whether the student performs community service work, and the student’s gender)
- How can you find how many girls were surveyed in all? (Add the number who perform community service to the number who do not: \(30 + 10 = 40\).)
- How can you find how many boys were surveyed in all? (Add the number who perform community service to the number who do not, or subtract the number of girls from the total number of students surveyed: \(60 - 40 = 20\).)
- How do you expect the difference between the numbers of boys and girls might affect the frequency of the data in each category? (There are twice as many girls as boys, so the frequencies should be about twice as great for the girls.)

During Problem 5.1 B, ask:

- How many girls did Alejandra survey this time? (15)
- How many boys did she survey? (45)
• How do you expect that the difference in the ratios of girls to boys between the two surveys will affect the frequencies in the data? (The frequencies for the boys probably will be much higher than the girls’ frequencies in the second survey.)

**Problem 5.2**

Before Problem 5.2, ask: How are the categories of the surveys in this problem different than those in Problem 5.1? (There are 4 categories for one variable in this problem, and there were only 2 categories for each variable in Problem 5.1.)

During Problem 5.2 B, ask:

• What numbers in the table of survey results were used to find the relative frequency of 35% in the table for Part B? (The number of teens who enjoyed comedies, 42, was divided by the frequency of teens, 120.)

• What numbers in the table of survey results were used to find the relative frequency of 10% in the table for Part B? (The number of parents who liked their teens watching comedies, 3, was divided by the frequency of parents, 30.)

• What will the sum of the percentages in each row of the relative frequency table be? (100%)

During Problem 5.2 C, ask:

• What numbers in the table of survey results were used to find the relative frequency of 93.3% in the table for Part C? (The number of teens who enjoyed comedies, 42, was divided by the frequency of people who preferred comedies, 45.)

• What numbers in the table of survey results were used to find the relative frequency of 6.7% in the table for Part C? (The number of parents who liked their teens watching comedies, 3, was divided by the frequency of people who preferred comedies, 45.)

• What will the sum of the percentages in each column of the relative frequency table be? (100%)

After Problem 5.2, ask: How do you know which relative frequency table to use when comparing the data? (Use the table in Part B to compare movie-type preferences within an age group, and use the table in Part C to compare age groups’ preferences for each movie type.)

**Summarize**

To summarize the lesson, ask:

• What makes a set of data categorical? (The data are separable into distinct groups that are mutually exclusive.)

• What makes a set of data bivariate? (It involves two different variables.)

• How do you find the relative frequency for a data value in a two-way table? (Divide the value by the frequency for that row or column.)

• When is a relative frequency table most helpful in describing a set of bivariate categorical data? (when the data are not evenly distributed among the categories)
Assignment Guide for Investigation 5

Problem 5.1, Exercises 1, 3
Problem 5.2, Exercises 2, 4

Answers to Investigation 5

Problem 5.1

A. 1. See Figure 1.
   2. $30 \div 40 = 0.75 = 75\%$
   3. $15 \div 20 = 0.75 = 75\%$
   4. The relative frequencies are equal. This shows that a student’s gender does not affect whether or not the student performs community service.
   5. Because the relative frequencies are equal, boys are just as likely to perform community service as girls. Twice as many girls were surveyed as boys, so the frequency of girls is twice that of boys.

B. 1. See Figure 2.
   2. boys: $30 \div 45 = 0.67 = 67\%$; girls: $10 \div 15 = 0.67 = 67\%$
   3. The relative frequencies are equal for boys and girls, just as in Part A, so boys from the second group of students surveyed are just as likely to perform community service as the girls in that group. The relative frequency in this group is lower than in Part A, so a student in the second group is less likely to perform community service than a student in the first group.

Figure 1:

<table>
<thead>
<tr>
<th>Gender</th>
<th></th>
<th></th>
<th>Frequency</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>girl</td>
<td>boy</td>
<td></td>
</tr>
<tr>
<td>Community Service</td>
<td>30</td>
<td>15</td>
<td>45</td>
</tr>
<tr>
<td>no</td>
<td>10</td>
<td>5</td>
<td>15</td>
</tr>
<tr>
<td>Frequency</td>
<td>40</td>
<td>20</td>
<td></td>
</tr>
</tbody>
</table>

Figure 2:

<table>
<thead>
<tr>
<th>Gender</th>
<th></th>
<th></th>
<th>Frequency</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>girl</td>
<td>boy</td>
<td></td>
</tr>
<tr>
<td>Community Service</td>
<td>10</td>
<td>30</td>
<td>40</td>
</tr>
<tr>
<td>no</td>
<td>5</td>
<td>15</td>
<td>20</td>
</tr>
<tr>
<td>Frequency</td>
<td>15</td>
<td>45</td>
<td></td>
</tr>
</tbody>
</table>
Problem 5.2

A. 1. See Figure 3.
2. Four times as many teens as parents were surveyed. The most popular type of movie among teens and parents combined is comedy, and the least popular type is documentary.

B. See Figure 4.
1. Teens prefer comedies most, and parents prefer documentaries most.
2. Because there were many more teens surveyed than parents, comparing frequencies does not appropriately represent the data. Using relative frequencies, 47% of parents prefer documentaries, while only 12% of teens prefer that movie type.

C. See Figure 5.
If a respondent replied that they preferred comedy or action movies, the chances are much greater that the respondent is a teen than a parent. If their favorite type of movie was a documentary, then it is equally likely that they are a teen or a parent.

---

**Figure 3:**

<table>
<thead>
<tr>
<th>Age Group</th>
<th>Type of Movie</th>
<th>Frequency</th>
</tr>
</thead>
<tbody>
<tr>
<td>teen</td>
<td>comedy: 42</td>
<td>romance: 28</td>
</tr>
<tr>
<td>parent</td>
<td>3</td>
<td>11</td>
</tr>
<tr>
<td>Frequency</td>
<td>45</td>
<td>39</td>
</tr>
</tbody>
</table>

**Figure 4:**

<table>
<thead>
<tr>
<th>Age Group</th>
<th>Comedy</th>
<th>Romance</th>
<th>Documentary</th>
<th>Action</th>
</tr>
</thead>
<tbody>
<tr>
<td>teen</td>
<td>35%</td>
<td>23%</td>
<td>12%</td>
<td>30%</td>
</tr>
<tr>
<td>parent</td>
<td>10%</td>
<td>37%</td>
<td>47%</td>
<td>7%</td>
</tr>
</tbody>
</table>

**Figure 5:**

<table>
<thead>
<tr>
<th>Age Group</th>
<th>Comedy</th>
<th>Romance</th>
<th>Documentary</th>
<th>Action</th>
</tr>
</thead>
<tbody>
<tr>
<td>teen</td>
<td>93%</td>
<td>72%</td>
<td>50%</td>
<td>95%</td>
</tr>
<tr>
<td>parent</td>
<td>7%</td>
<td>28%</td>
<td>50%</td>
<td>5%</td>
</tr>
</tbody>
</table>
**Exercises**

1. a. See Figure 6.
   
   b. 80%
   
   c. 70%
   
   d. A girl is more likely than a boy to enjoy playing volleyball.

2. a. See Figure 7.
   
   b. See Figure 8.

   c. Among voters who support the tax increase, Martinez is heavily favored. Among voters who do not support the tax increase, there is no clear favorite, though Martinez is least favored.

3. B

4. a. See Figure 9.
   
   b. fruit
   
   c. candy

**Figure 6:**

<table>
<thead>
<tr>
<th>Gender</th>
<th>Enjoy Playing Volleyball</th>
<th>Frequency</th>
</tr>
</thead>
<tbody>
<tr>
<td>girl</td>
<td>yes</td>
<td>16</td>
</tr>
<tr>
<td></td>
<td></td>
<td>21</td>
</tr>
<tr>
<td></td>
<td></td>
<td>37</td>
</tr>
<tr>
<td>boy</td>
<td>no</td>
<td>4</td>
</tr>
<tr>
<td></td>
<td></td>
<td>9</td>
</tr>
<tr>
<td></td>
<td></td>
<td>13</td>
</tr>
<tr>
<td></td>
<td>Frequency</td>
<td>20</td>
</tr>
<tr>
<td></td>
<td></td>
<td>30</td>
</tr>
</tbody>
</table>

**Figure 7:**

<table>
<thead>
<tr>
<th>Candidate</th>
<th>Support Tax</th>
<th>Wilkins</th>
<th>Block</th>
<th>Martinez</th>
<th>O’Neil</th>
<th>Frequency</th>
</tr>
</thead>
<tbody>
<tr>
<td>yes</td>
<td></td>
<td>5</td>
<td>3</td>
<td>28</td>
<td>1</td>
<td>37</td>
</tr>
<tr>
<td>no</td>
<td></td>
<td>18</td>
<td>16</td>
<td>10</td>
<td>19</td>
<td>63</td>
</tr>
<tr>
<td>Frequency</td>
<td></td>
<td>23</td>
<td>19</td>
<td>38</td>
<td>20</td>
<td></td>
</tr>
</tbody>
</table>

**Figure 8:**

<table>
<thead>
<tr>
<th>Candidate</th>
<th>Support Tax</th>
<th>Wilkins</th>
<th>Block</th>
<th>Martinez</th>
<th>O’Neil</th>
</tr>
</thead>
<tbody>
<tr>
<td>yes</td>
<td></td>
<td>14%</td>
<td>8%</td>
<td>76%</td>
<td>3%</td>
</tr>
<tr>
<td>no</td>
<td></td>
<td>29%</td>
<td>25%</td>
<td>16%</td>
<td>30%</td>
</tr>
</tbody>
</table>

**Figure 9:**

<table>
<thead>
<tr>
<th>Favorite Snack</th>
<th>Exercise Regularly</th>
<th>Frequency</th>
</tr>
</thead>
<tbody>
<tr>
<td>chips</td>
<td>yes</td>
<td>8</td>
</tr>
<tr>
<td>candy</td>
<td></td>
<td>4</td>
</tr>
<tr>
<td>fruit</td>
<td></td>
<td>28</td>
</tr>
<tr>
<td>nuts</td>
<td>no</td>
<td>80</td>
</tr>
<tr>
<td></td>
<td></td>
<td>92</td>
</tr>
<tr>
<td></td>
<td></td>
<td>30</td>
</tr>
<tr>
<td></td>
<td></td>
<td>18</td>
</tr>
<tr>
<td></td>
<td>Frequency</td>
<td>88</td>
</tr>
<tr>
<td></td>
<td></td>
<td>96</td>
</tr>
<tr>
<td></td>
<td></td>
<td>58</td>
</tr>
<tr>
<td></td>
<td></td>
<td>28</td>
</tr>
</tbody>
</table>
d. See Figure 10.
e. fruit
f. No; while the frequency for those who do not exercise is greater, 48 to 38, the relative frequency is greater for those who do exercise, 76% to 22%. The relative frequencies show that people who do exercise regularly prefer healthier snacks than those who do not exercise regularly.

**Figure 10:**

<table>
<thead>
<tr>
<th>Favorite Snack</th>
<th>chips</th>
<th>candy</th>
<th>fruit</th>
<th>nuts</th>
</tr>
</thead>
<tbody>
<tr>
<td>Exercise</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>yes</td>
<td>16%</td>
<td>8%</td>
<td>56%</td>
<td>20%</td>
</tr>
<tr>
<td>no</td>
<td>36%</td>
<td>42%</td>
<td>14%</td>
<td>8%</td>
</tr>
<tr>
<td>Regularly</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Alison surveys 50 students at her school and asks whether they have a home computer, and whether they have a cell phone. She records each student’s answers. Of the 40 students whose family has a computer, 30 have a cell phone. A total of 32 students that Alison surveyed have cell phones.

1. Copy and complete the table to show these data. Find each frequency by adding the numbers in that row or column.

<table>
<thead>
<tr>
<th>Cell Phone</th>
<th>Frequency</th>
</tr>
</thead>
<tbody>
<tr>
<td>yes</td>
<td></td>
</tr>
<tr>
<td>no</td>
<td></td>
</tr>
</tbody>
</table>

2. What do the frequencies tell you about Alison’s data?

3. Copy and complete the table to show the relative frequency of cell phone owners based on whether the family owns a computer.

<table>
<thead>
<tr>
<th>Cell Phone</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>yes</td>
<td>75%</td>
</tr>
<tr>
<td>no</td>
<td></td>
</tr>
</tbody>
</table>

a. What do the relative frequencies tell you about whether having a home computer is related to whether the student has a cell phone?

b. Ben looks at the numbers of students without cell phones and concludes that students are more likely not to have a cell phone if their families have computers. Explain Ben’s error.

c. Do the relative frequencies give a different impression than the actual numbers?

4. Make a table to show the relative frequency of home computers based on whether the student has a cell phone. Explain what this shows about the data.
For Exercises 1–3, rewrite each table of values as a table of relative frequencies compared to the entire population. Round relative frequencies to the nearest whole percent.

1. **Have Pets**

<table>
<thead>
<tr>
<th>Have Siblings</th>
<th>yes</th>
<th>no</th>
</tr>
</thead>
<tbody>
<tr>
<td>yes</td>
<td>32</td>
<td>22</td>
</tr>
<tr>
<td>no</td>
<td>27</td>
<td>19</td>
</tr>
</tbody>
</table>

2. **Club Membership**

<table>
<thead>
<tr>
<th>Math Ability</th>
<th>Spanish</th>
<th>Drama</th>
<th>Science</th>
</tr>
</thead>
<tbody>
<tr>
<td>low</td>
<td>10</td>
<td>18</td>
<td>6</td>
</tr>
<tr>
<td>high</td>
<td>12</td>
<td>14</td>
<td>20</td>
</tr>
</tbody>
</table>

3. **Income**

<table>
<thead>
<tr>
<th>Job Satisfaction</th>
<th>low</th>
<th>medium</th>
<th>high</th>
</tr>
</thead>
<tbody>
<tr>
<td>low</td>
<td>10</td>
<td>8</td>
<td>8</td>
</tr>
<tr>
<td>medium</td>
<td>9</td>
<td>12</td>
<td>8</td>
</tr>
<tr>
<td>high</td>
<td>6</td>
<td>30</td>
<td>9</td>
</tr>
</tbody>
</table>

For Exercises 4–6, make one relative frequency table that shows relative frequencies by row, and a second relative frequency table that shows relative frequencies by column. Round relative frequencies to the nearest whole percent.

4. **Gender**

<table>
<thead>
<tr>
<th>Have a Summer Job</th>
<th>boy</th>
<th>girl</th>
</tr>
</thead>
<tbody>
<tr>
<td>yes</td>
<td>12</td>
<td>9</td>
</tr>
<tr>
<td>no</td>
<td>20</td>
<td>15</td>
</tr>
</tbody>
</table>

5. **Grade**

<table>
<thead>
<tr>
<th>Play Sport</th>
<th>6</th>
<th>7</th>
<th>8</th>
</tr>
</thead>
<tbody>
<tr>
<td>yes</td>
<td>17</td>
<td>11</td>
<td>14</td>
</tr>
<tr>
<td>no</td>
<td>23</td>
<td>29</td>
<td>26</td>
</tr>
</tbody>
</table>

6. **Transportation to School**

<table>
<thead>
<tr>
<th>Sibling</th>
<th>bus</th>
<th>walk/bike</th>
<th>car</th>
</tr>
</thead>
<tbody>
<tr>
<td>none</td>
<td>9</td>
<td>2</td>
<td>4</td>
</tr>
<tr>
<td>same school</td>
<td>12</td>
<td>4</td>
<td>4</td>
</tr>
<tr>
<td>different school</td>
<td>22</td>
<td>4</td>
<td>4</td>
</tr>
</tbody>
</table>
1. Rama asks 45 students whom they like for class president. She records the candidate’s name and the student’s gender.

<table>
<thead>
<tr>
<th>Gender</th>
<th>Julia</th>
<th>Edie</th>
<th>Bill</th>
<th>Frequency</th>
</tr>
</thead>
<tbody>
<tr>
<td>girl</td>
<td>13</td>
<td>12</td>
<td>5</td>
<td></td>
</tr>
<tr>
<td>boy</td>
<td>2</td>
<td>3</td>
<td>10</td>
<td></td>
</tr>
</tbody>
</table>

a. Complete the table. What do the frequencies tell you about Rama’s data?

b. Complete the table to show the relative frequency of each candidate based on the students’ gender. Explain what this tells you about Rama’s data.

<table>
<thead>
<tr>
<th>Gender</th>
<th>Candidate</th>
</tr>
</thead>
<tbody>
<tr>
<td>girl</td>
<td>43%</td>
</tr>
<tr>
<td>boy</td>
<td></td>
</tr>
</tbody>
</table>

c. Make a table to show for each candidate the relative frequency of students by gender. Explain what this shows about the data.

d. If the same numbers of boys and girls vote in the class president election, who do you predict will win? Explain your reasoning.
2. Antoine wants to find out whether the amount of time students spend online affects their grades. He asks 80 students whether they spend a little time, a medium amount of time, or a lot of time online. He asks the same students whether or not they get good grades at school.

<table>
<thead>
<tr>
<th>Good Grades?</th>
<th>little</th>
<th>medium</th>
<th>a lot</th>
<th>Frequency</th>
</tr>
</thead>
<tbody>
<tr>
<td>yes</td>
<td>16</td>
<td>24</td>
<td>10</td>
<td></td>
</tr>
<tr>
<td>no</td>
<td>7</td>
<td>13</td>
<td>10</td>
<td></td>
</tr>
</tbody>
</table>

a. Complete the frequency row and column in the table.

b. What do the frequencies tell you about Antoine’s data?

c. Complete the table to show the relative frequency of grades based on the amount of time spent online.

<table>
<thead>
<tr>
<th>Time Online</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
</tr>
<tr>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Good Grades?</th>
<th>little</th>
<th>medium</th>
<th>a lot</th>
</tr>
</thead>
<tbody>
<tr>
<td>yes</td>
<td>32%</td>
<td></td>
<td></td>
</tr>
<tr>
<td>no</td>
<td>23%</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

d. What do the relative frequencies tell you about how the amount of time online varies between the groups of students?

e. Marta sees that there are 10 students with good grades and 10 students without good grades who spend a lot of time online and concludes that spending a lot of time online does not affect grades. Explain Marta’s error.

f. Do the relative frequencies of the students who spend a medium amount of time online give a different impression about the data than the actual numbers? Explain.
Data that are categorical are separable into distinct groups that are mutually exclusive. Categorical data can be gathered by separating objects into groups based on some common property and then counting the number of objects in each group. Examples of categorical data include gender, age, and whether or not a person has any siblings.

Alejandra wants to determine how a student’s participation in community service might be related to that student’s gender.

**A.** Alejandra asks 60 students whether they perform any community service work. She records each student’s gender and answer. There are 30 girls who perform community service and 10 girls who do not. There are 15 boys who perform community service and 5 boys who do not.

1. Copy and complete the two-way table by filling in the data from Alejandra’s survey. Find each frequency by adding the numbers in that row or column.

<table>
<thead>
<tr>
<th>Gender</th>
<th>Community Service</th>
<th>Frequency</th>
</tr>
</thead>
<tbody>
<tr>
<td>girl</td>
<td>yes</td>
<td></td>
</tr>
<tr>
<td>girl</td>
<td>no</td>
<td></td>
</tr>
<tr>
<td>boy</td>
<td>yes</td>
<td></td>
</tr>
<tr>
<td>boy</td>
<td>no</td>
<td></td>
</tr>
</tbody>
</table>

2. Find the relative frequency of girls who perform community service by dividing the number of girls who perform community service by the total number of girls. Write the relative frequency as a percent.

3. Find the relative frequency of boys who perform community service by dividing the number of boys who perform community service by the total number of boys. Write the relative frequency as a percent.

4. Compare the relative frequencies you found in Parts 2 and 3. Explain what this tells you about the data.

5. Marcus looks at the data and says that girls are twice as likely to perform community service as boys because \(30 \div 15 = 2\). Explain Marcus’ error.
B. Alejandra asks another 60 students about community service. Of the 15 girls she surveyed, 10 perform community service. Three times as many boys perform community service.

1. Make a two-way table to display Alejandra’s new data.
2. Find the relative frequencies of boys and girls who perform community service.
3. Compare the relative frequencies to those you found in Part A. What does that comparison tell you about the two data sets that look different?

Alejandra wants to find out whether the types of movies that teens enjoy watching are the same types of movies that their parents enjoy.

<table>
<thead>
<tr>
<th>Age Group</th>
<th>Comedy</th>
<th>Romance</th>
<th>Documentary</th>
<th>Action</th>
<th>Frequency</th>
</tr>
</thead>
<tbody>
<tr>
<td>teen</td>
<td>42</td>
<td>28</td>
<td>14</td>
<td>36</td>
<td></td>
</tr>
<tr>
<td>parent</td>
<td>3</td>
<td>11</td>
<td>14</td>
<td>2</td>
<td></td>
</tr>
</tbody>
</table>

A. Copy the table above.

1. Find the frequency for each age group by adding the numbers in that row. Then find the frequencies for each column.
2. What do these frequencies tell you about the data?

B. Copy and complete the table below to show the relative frequency of each movie type for the age group.

<table>
<thead>
<tr>
<th>Type of Movie</th>
<th>Comedy</th>
<th>Romance</th>
<th>Documentary</th>
<th>Action</th>
</tr>
</thead>
<tbody>
<tr>
<td>Age Group</td>
<td>teen</td>
<td>parent</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>35%</td>
<td>10%</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

1. What do the relative frequencies tell you about the movies that are preferred by teens and parents?
2. Sheila sees that there are 14 teens and 14 parents who prefer documentaries and concludes that those movies are equally popular with the two groups. Explain Sheila’s error.
3. Do the relative frequencies of romance movies give a different impression about the data than the actual numbers?

C. Make a table to show the relative frequency of each age group for the movie type. Explain what this shows about the data.
Exercises

1. Isaiah asks 50 students whether they enjoy playing volleyball. Of the 20 girls he surveys, 16 enjoy volleyball. Of the boys he surveys, 21 enjoy volleyball.

   a. Copy and complete the table to show these data.

<table>
<thead>
<tr>
<th>Gender</th>
<th>Enjoy Playing Volleyball</th>
<th>Frequency</th>
</tr>
</thead>
<tbody>
<tr>
<td>girl</td>
<td>yes</td>
<td></td>
</tr>
<tr>
<td></td>
<td>no</td>
<td></td>
</tr>
<tr>
<td>boy</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

   b. What is the relative frequency of girls who enjoy volleyball?
   c. What is the relative frequency of boys who enjoy volleyball?
   d. What do the relative frequencies tell you about whether a student’s gender is related to whether he or she enjoys playing volleyball?

2. Penn asks 100 registered voters whom they intend to vote for in the upcoming election, and whether they support a tax increase that will help the schools. He finds that among the voters who support the tax increase, 5 will vote for Wilkins, 3 for Block, 28 for Martinez, and 1 for O’Neil. Among the voters who oppose the tax increase, 18 will vote for Wilkins, 16 for Block, 10 for Martinez, and 19 for O’Neil.

   a. Copy and complete the table to show these data.

<table>
<thead>
<tr>
<th>Candidate</th>
<th>Wilkins</th>
<th>Block</th>
<th>Martinez</th>
<th>O’Neil</th>
<th>Frequency</th>
</tr>
</thead>
<tbody>
<tr>
<td>Support Tax</td>
<td>yes</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>no</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

   b. Copy and complete the table to show the relative frequencies of each candidate based on whether the voter supports or opposes the tax increase.

<table>
<thead>
<tr>
<th>Candidate</th>
<th>Wilkins</th>
<th>Block</th>
<th>Martinez</th>
<th>O’Neil</th>
</tr>
</thead>
<tbody>
<tr>
<td>Support Tax</td>
<td>yes</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>no</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

   c. What do the relative frequencies tell you about how the candidates’ popularity is related to a voter’s opinion about the tax increase?
3. **Multiple Choice** What is the relative frequency of commuters with cars who ride their bikes to work?

   A. 0.03  
   B. 0.04  
   C. 0.14  
   D. 0.96

4. The table shows the results of a survey that asked people to name their favorite snack and whether or not they exercise regularly.

<table>
<thead>
<tr>
<th></th>
<th>Have a Car</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>yes</td>
</tr>
<tr>
<td>Ride Bike to Work</td>
<td>3</td>
</tr>
<tr>
<td>no</td>
<td>67</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Favorite Snack</th>
<th>Frequency</th>
</tr>
</thead>
<tbody>
<tr>
<td>chips</td>
<td>candy</td>
</tr>
<tr>
<td>-------</td>
<td>-------</td>
</tr>
<tr>
<td>yes</td>
<td>8</td>
</tr>
<tr>
<td>no</td>
<td>80</td>
</tr>
</tbody>
</table>

   a. Copy and complete the table by finding the frequencies for all rows and columns.
   b. Which snack was most popular among people who exercise regularly?
   c. Which snack was most popular among people who do not exercise regularly?
   d. Copy and complete the table to show the relative frequencies of each snack based on whether the person exercised regularly or not.

<table>
<thead>
<tr>
<th>Favorite Snack</th>
<th>Frequency</th>
</tr>
</thead>
<tbody>
<tr>
<td>chips</td>
<td>candy</td>
</tr>
<tr>
<td>-------</td>
<td>-------</td>
</tr>
<tr>
<td>yes</td>
<td>16%</td>
</tr>
<tr>
<td>no</td>
<td>36%</td>
</tr>
</tbody>
</table>

e. Which snack had the greatest difference in relative frequencies between people who exercise regularly and those who do not?

f. Of people who do not exercise regularly, 48 prefer fruit or nuts. Only 38 people who do exercise regularly prefer fruit or nuts. Does this mean that those who don’t exercise regularly prefer more healthy snacks than those who do exercise regularly? Explain why or why not.
CC Investigation 1 Answers to Additional Practice, Skill Practice, and Check-Up

Investigation 1 Additional Practice

1. a. \(\frac{1}{256} \cdot \left(\frac{1}{4}\right)^4 = \frac{1}{4} \times \frac{1}{4} \times \frac{1}{4} \times \frac{1}{4} = \frac{1}{256}\)
   
   b. Yes, \(\left(\frac{1}{4}\right)^6 = \frac{1}{4} \times \frac{1}{4} \times \frac{1}{4} \times \frac{1}{4} \times \frac{1}{4} \times \frac{1}{4}\)
      
      \[= \frac{1}{4,096} \times \frac{1}{4,096} < \frac{1}{4,000}\]
   
   c. No; \(\left(\frac{1}{5}\right)^4 = \frac{1}{5} \times \frac{1}{5} \times \frac{1}{5} \times \frac{1}{5} = \frac{1}{625}\)
      
      \[\frac{1}{625} > \frac{1}{4,000}\]

2. a. 4 cartons; \(s = \sqrt[3]{125} = 5\) ft; \(23 \div 5 = 4.6\)
   
   b. 6 cartons; \(s = \sqrt[3]{216} = 6\) ft;
      
      \(40 \div 6 = \text{about 6.7}\)
   
   c. No; \(s = \sqrt[3]{64} = 4\) ft; \(23 \div 4 = 5.75, \text{so}\)
      
      5 cartons will fit in each row; \(5 \times 3 = 15; 15 < 16\)
   
   d. 15 ft; \(s = \sqrt[3]{27} = 3\) ft; \(20 \div 4 = 5\) rows;
      
      \(5 \times 3 = 15\) ft

Skill: Exponents with Whole Numbers and Decimals

1. 512
2. 32
3. 625
4. 1,000
5. 0.064
6. about 39
7. about 10.5
8. about 238
9. 243
10. 2,401
11. about 118
12. 0.0625

Skill: Exponents with Fractions

13. \(\frac{1}{16}\)
14. \(\frac{1}{8}\)
15. \(\frac{16}{81}\)
16. \(\frac{125}{512}\)
17. \(\frac{1}{1296}\)
18. \(\frac{256}{2,401}\)
19. 27
20. \(\frac{8}{125}\)
21. \(\frac{9}{16}\)
22. 343
23. \(\frac{1}{4,096}\)
24. \(\frac{16}{81}\)
CC Investigation 1 Answers to Additional Practice, Skill Practice, and Check-Up (continued)

Investigation 1 Check-Up

<table>
<thead>
<tr>
<th>Negative Exponent</th>
<th>Positive Exponent</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$3^{-2}$</td>
<td>$\left(\frac{1}{3}\right)^2$</td>
<td>$\frac{1}{9}$</td>
</tr>
<tr>
<td>$4^{-5}$</td>
<td>$\left(\frac{1}{4}\right)^5$</td>
<td>$\frac{1}{1024}$</td>
</tr>
<tr>
<td>$2^{-4}$</td>
<td>$\left(\frac{1}{2}\right)^4$</td>
<td>$\frac{1}{16}$</td>
</tr>
<tr>
<td>$7^{-2}$</td>
<td>$\left(\frac{1}{7}\right)^2$</td>
<td>$\frac{1}{49}$</td>
</tr>
<tr>
<td>$8^{-2}$</td>
<td>$\left(\frac{1}{8}\right)^2$</td>
<td>$\frac{1}{64}$</td>
</tr>
<tr>
<td>$\left(\frac{1}{4}\right)^{-2}$</td>
<td>$4^2$</td>
<td>$16$</td>
</tr>
<tr>
<td>$\left(\frac{1}{3}\right)^{-3}$</td>
<td>$3^3$</td>
<td>$27$</td>
</tr>
<tr>
<td>$\left(\frac{1}{2}\right)^{-5}$</td>
<td>$2^5$</td>
<td>$32$</td>
</tr>
<tr>
<td>$\left(\frac{1}{5}\right)^{-4}$</td>
<td>$5^4$</td>
<td>$625$</td>
</tr>
</tbody>
</table>

2. a. $4 \times 4 \times 4 = 4^3 = 64$ messages  
   b. 5 rounds; $4^3 = 64; 4^4 = 256; 4^5 = 1,024$
3. a. B: $V = s^3; 512 = s^3; s = \sqrt[3]{512} = 8$ in.;  
   D: $V = s^3; 64 = s^3; s = \sqrt[3]{64} = 4$ in.  
   b. A: $V = \frac{2}{3}\pi r^3; 18\pi = \frac{2}{3}\pi r^3; 27 = r^3;  
   r = \sqrt[3]{27}; r = 3$ in.;  
   C: $V = \frac{2}{3}\pi r^3; 144\pi = \frac{2}{3}\pi r^3; 216 = r^3; r = \sqrt[3]{216};  
   r = 6$ in.  
   c. about 7 in.;  
   $V = \frac{3}{4} \times 144\pi = 108\pi \approx 339; V = s^3; s \approx \sqrt[3]{339}; \text{ about 7 in.}$  
   d. about 9 in.; $3 \times 512 = 1,536$ in.$^3;$  
   $V = \frac{2}{3}\pi r^3; 1,536 = \frac{2}{3}\pi r^3; 733 = r^3;  
   r \approx \sqrt[3]{733}; \text{ about 9 in.}$
Investigation 2 Additional Practice

1. The graphs are straight lines, so the trains are traveling at constant speeds.
2. Find the slope of the line, or find the distance at 1 hour; 60 mi/h.
3. Train A was 60 miles from Omaha at time = 0, when Train B left Omaha.
4. \( d = 60t \)
5. \( d = 60t + 60 \)
6. The equation is in the form \( y = mx + b \), where \( m \) represents the slope. In the equation \( d = 60t + 60 \), the slope, \( m \), is 60. The slope is the unit rate. Train A’s speed is 60 mi/h.
7. Only one; the equation is a function.
8. a. \( 60t = 75(t – 1) \)
   b. the time, \( t \), when trains B and C are at the same location
   c. \( 60t = 75(t – 1); 60t = 75t – 75; -15t = -75; t = 5 \); There is one solution.
   d. Train C will catch train B 5 hours after train B left Omaha.

Skill: Finding Unit Rate

1. 2 ft/sec
2. –1 L/min
3. $2.50/lb
4. 3 cm/day
5. 60 mi/h
6. 37°F/min
7. 14.5
8. \( \frac{1}{2} \)

Skill: Solutions to Equations

9. one solution
10. no solution
11. no solution
12. one solution
13. one solution
14. infinitely-many solutions
15. no solution
16. infinitely-many solutions
17. one solution
18. infinitely-many solutions

Investigation 2 Check-Up

1. a. The graph is a straight line, so look at the volume at time = 1 sec; the unit rate is 20 fl oz/sec.
   b. \( v = 25t \) is in the form \( y = mx \), where \( m \) represents the slope or unit rate of the equation; \( m = 25 \); the unit rate is 25 fl oz/sec.
   c. Find the unit rate at each time by dividing the volume by the time:
      \( 32 ÷ 2 = 16; 56 ÷ 3.5 = 16; 80 ÷ 5 = 16; 104 ÷ 6.5 = 16 \); the unit rate is 16 fl oz/sec.
   d. She filled the second jug the fastest since the filling rate was the greatest: 25 > 20 > 16.

2. a. 

<table>
<thead>
<tr>
<th>Time, ( t ) (sec)</th>
<th>Distance, ( d ) (ft)</th>
</tr>
</thead>
<tbody>
<tr>
<td>3</td>
<td>15</td>
</tr>
<tr>
<td>4.5</td>
<td>22.5</td>
</tr>
<tr>
<td>7</td>
<td>35</td>
</tr>
<tr>
<td>10.5</td>
<td>52.5</td>
</tr>
</tbody>
</table>

b. Only one; the equation is a function.

<table>
<thead>
<tr>
<th>Distance, ( d )</th>
</tr>
</thead>
<tbody>
<tr>
<td>15</td>
</tr>
<tr>
<td>22.5</td>
</tr>
<tr>
<td>35</td>
</tr>
<tr>
<td>52.5</td>
</tr>
</tbody>
</table>

c. Yes, she will catch him 30 seconds after he left; \( 6(t – 5) = 5t; 6t – 30 = 5t; t = 30 \).

d. No, the solution is a negative number, which does not make sense in this situation; \( 4.5(t – 8) = 5t; 4.5t – 36 = 5t; 0.5t = –36; t = –72 \).

e. They will stay together, since they both are traveling at the same speed and leave at the same time.
Investigation 3 Additional Practice

1. a. $A'(-1, 0), B'(3, -2), O'(-3, -4)$

b. $A'(-2, 4), B'(-6, 2), O'(0, 0)$

c. $F'(3, -2), G'(5, -1), O'(2, 0)$

d. $A'(4, -2), B'(2, -6), O'(0, 0)$

e. $F'(3, 4), G'(5, 3), O'(2, 2)$

f. $A'(1, 2), B'(3, 1), O'(0, 0)$
CC Investigation 3 Answers to Additional Practice, Skill Practice, and Check-Up (continued)

9. 

\[ F'(-3, -2), G'(-7, 0), O'(-1, 2) \]

2. a. 180° rotation with a center of rotation at the origin and dilation with scale factor of \( \frac{1}{2} \) and a center of dilation at the origin

b. \( A'(-2, -4), B'(-6, -2), O'(0, 0) \)

c. They are similar.

Skill: Identify Transformations

1. rotation
2. translation
3. reflection or rotation
4. dilation
5. reflection
6. dilation

Skill: Transformations on the Coordinate Plane

7. \( A'(1, 7), B'(-1, 4), C'(4, 4) \)
8. \( A'(-1, 2), B'(-3, -1), C'(2, -1) \)
9. \( A'(1, -4), B'(-1, -1), C'(4, -1) \)
10. \( A'(-3, 4), B'(-1, 1), C'(-6, 1) \)
11. \( A'(4, -1), B'(1, 1), C'(1, -4) \)
12. \( A'(1, -2), B'(4, -4), C'(4, 1) \)
13. \( A'(2, 8), B'(-2, 2), C'(8, 2) \)
14. \( A'(1, 4), B'(0, 2.5), C'(2.5, 2.5) \)
Investigation 3 Check-Up
1. a. $P'(-2, -1), C'(3, -1), R'(-4, -3), W'(5, -3)$

b. The image of the line is the same as the original figure, though points $P$ and $C$ have been translated 3 units to the right.

c. $P'(-2, -3), C'(3, -3), R'(-4, -1), W'(5, -1)$

d. $P'(3, 8), C'(3, 3), R'(1, 10), W'(1, 1)$

e. Line $P'C'$ would be in the same position, but $P$ would be at $(3, -2)$ and line $R'W'$ would be 2 units to the right of line $P'C'$.

f. They remain parallel after each transformation.

2. Line segment $PC$ was transformed in the same way as line $PC$ each time.
CC Investigation 3 Answers to Additional Practice, Skill Practice, and Check-Up (continued)

3. a. $A' (6, 5), B' (4, 3), C' (4, 11), D' (6, 9)$

b. $A' (-1, 2), B' (-3, 4), C' (5, 4), D' (3, 2)$

c. $A' (0, 1), B' (-2, 3), C' (-2, -5), D' (0, -3)$

d. $A' (1, 0.5), B' (2, 1.5), C' (2, -2.5), D' (1, -1.5)$

e. $A' (2, 1), B' (6, 5), C' (6, -11), D' (2, -7)$

4. a. $A'' (-2, 4), B'' (0, 2), C'' (-7, 2), D'' (-5, 4)$

b. $A'' (-1.5, 0), B'' (-0.5, 1), C'' (-0.5, -3), D'' (-1.5, -2)$

c. The figures are similar. In Part (a), the figures are congruent; in Part (b), they are similar.
Investigation 4 Additional Practice

1. a. ∠c is congruent to ∠g, and ∠g forms a supplementary pair with ∠f, so
   \( m\angle c + m\angle f = 180 \); \( 40 + 144 \neq 180 \), so at least one of the measures must be incorrect.

   \( b. \) 18N must be perpendicular to 2W and 4W; since \( ∠k \) and \( ∠n \) are both supplementary and congruent, they must each measure 90°.

   \( c. \) They are similar. Both triangles include \( ∠a \) and a 90° angle, so two pairs of corresponding angles are congruent, and the triangles are similar.

   \( d. \) \( m\angle g = 40° \); \( ∠c \) and \( ∠g \) are corresponding angles on a transversal, so their measures are equal.

2. a. Scoop C would be best since it has the greatest volume:
   \[ V_A = \pi r^2 h = \pi(5)^2(3) = 75\pi \text{ cm}^3; \]
   \[ V_B = \pi r^2 h = \pi(2.5)^2(6) = 37.5\pi \text{ cm}^3; \]
   \[ V_C = \frac{1}{3}\pi r^2 h = \frac{1}{3}\pi(5)^2(10) = 83\pi \text{ cm}^3; \]
   \[ V_D = \frac{1}{3}\pi r^2 h = \frac{1}{3}\pi(6)^2(6) = 72\pi \text{ cm}^3; \]
   \[ 83\pi > 75\pi > 72\pi > 37.5\pi \]

   \( b. \) \( 10 \times (83\pi - 75\pi) = 10 \times 8\pi = 80\pi \text{ cm}^3 \)

Skill: Measure Angles

1. 70°
2. 140°
3. 40°
4. 70°
5. 110°
6. 140°
7. 70°
8. 110°

Skill: Evaluate Expressions

9. 120
10. 45
11. 180
12. 36
13. 6.25
14. 0.25
15. 512
16. 74.09
17. 0.42
18. 6
19. 5.3
20. 10.67
21. 26.27
22. 12.57
23. 41.78
24. 439.82
Investigation 4 Check-Up

1. a. $a$ and $g$, $b$ and $h$, $c$ and $e$, $d$ and $f$
   b. $a$ and $c$, $b$ and $d$, $g$ and $e$, $h$ and $f$
   c. $a$ and $g$, $b$ and $h$
   d. $a$ and $e$, $d$ and $h$
   e. $a$ and $h$, $b$ and $g$, $c$ and $f$, $d$ and $e$
   f. $m\angle b = 40^\circ$; $m\angle c = 140^\circ$; $m\angle d = 40^\circ$; $m\angle e = 140^\circ$; $m\angle f = 40^\circ$; $m\angle g = 140^\circ$; $m\angle h = 40^\circ$

2. a. about 7.5 in.;
   \[ V = lwh = 3 \times 3 \times 7 = 63 \text{ in.}^3 \]
   \[ 63 \div 2 = 31.5 \text{ in.}^3 \]
   \[ V_{\text{sphere}} = \frac{4}{3}\pi r^3; \]
   \[ 31.5 = \frac{4}{3}\pi r^3; \]
   \[ r \approx \sqrt[3]{7.52} \approx 2 \text{ in.}; \]
   \[ V_{\text{cone}} = \frac{1}{3}\pi r^2 h; \]
   \[ 31.5 = \frac{1}{3}\pi (2)^2 h; \]
   \[ h \approx 7.5 \text{ in.}; \]
   b. about 5.4 in.;
   \[ V_{\text{cone}} = \frac{1}{3}\pi r^2 h; \]
   \[ 31.5 = \frac{1}{3}\pi r^2 (4); \]
   \[ r^2 \approx 7.5; \]
   \[ r \approx \sqrt{7.5} \approx 2.7 \text{ in.}; \]
   \[ 2 \times 2.7 = 5.4 \text{ in.}; \]

3. a. $m\angle 1 = 180 - 45 = 135^\circ$; the angles are supplementary; $m\angle 2 = m\angle 1 = 135^\circ$; the angles are corresponding; $m\angle 3 = 45^\circ$; the angle is an alternate exterior angle to the 45° angle; $m\angle 4 = 115^\circ$; the angle is an alternate exterior angle to the 115° angle; $m\angle 5 = 180 - 115 = 65^\circ$; the angles are supplementary.
   b. The upper left angle is vertical to a 115° angle, so its measure also is 115°; the upper right angle is supplementary to an angle measuring 45°, so its measure is $180 - 45 = 135^\circ$; the lower right angle is a corresponding angle to an angle measuring 45°, so its measure also is 45°.
   c. 45°; 180 - 115 = 65°;
   \[ 180 - (65 + 45) = 70^\circ \]

4. one pillar; $V_{\text{cylinder}} = \pi r^2 h = \pi (2.25)^2 (20) 
\approx 318 \text{ ft}^3; 1 \text{ yd}^3 = 27 \text{ ft}^3$,
   \[ V_{\text{cylinder}} \approx 318 \div 27 \approx 11.8 \text{ yd}^3; \]
   \[ 13.5 \div 11.8 \approx 1.14 \]
Investigation 5 Additional Practice

1. See Figure 1.

2. More families have home computers than do not, and more students have cell phones than do not.

3. See Figure 2.
   a. Students whose families have a home computer are much more likely to have a cell phone.
   b. Ben based his conclusion on the frequencies rather than the relative frequencies. The frequency of those without a cell phone, but with a home computer is higher because many more students with home computers than students without home computers were surveyed.
   c. Yes, the relative frequencies show that students with home computers are more likely to have a cell phone.

4. See Figure 3; Students with a cell phone almost always also have a home computer, but the frequency is about even among students without a cell phone.

---

**Figure 1:**

| Home Computer | Cell Phone
<table>
<thead>
<tr>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>yes</td>
</tr>
<tr>
<td>yes</td>
<td>30</td>
</tr>
<tr>
<td>no</td>
<td>2</td>
</tr>
<tr>
<td><strong>Frequency</strong></td>
<td>32</td>
</tr>
</tbody>
</table>

**Figure 2:**

| Home Computer | Cell Phone
<table>
<thead>
<tr>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>yes</td>
</tr>
<tr>
<td>yes</td>
<td>75%</td>
</tr>
<tr>
<td>no</td>
<td>20%</td>
</tr>
</tbody>
</table>

**Figure 3:**

| Home Computer | Cell Phone
<table>
<thead>
<tr>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>yes</td>
</tr>
<tr>
<td>yes</td>
<td>94%</td>
</tr>
<tr>
<td>no</td>
<td>6%</td>
</tr>
</tbody>
</table>
### CC Investigation 5 Answers to Additional Practice, Skill Practice, and Check-Up (continued)

**Skill: Calculating Relative Frequencies**

#### 1.

<table>
<thead>
<tr>
<th>Have Siblings</th>
<th>Have Pets</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>yes</td>
<td>32%</td>
<td>22%</td>
</tr>
<tr>
<td>no</td>
<td>27%</td>
<td>19%</td>
</tr>
</tbody>
</table>

#### 2.

<table>
<thead>
<tr>
<th>Math Ability</th>
<th>Club Membership</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>low</td>
<td>Spanish</td>
<td>Drama</td>
</tr>
<tr>
<td>low</td>
<td>13%</td>
<td>23%</td>
</tr>
<tr>
<td>high</td>
<td>15%</td>
<td>18%</td>
</tr>
</tbody>
</table>

#### 3.

<table>
<thead>
<tr>
<th>Job Satisfaction</th>
<th>Income</th>
</tr>
</thead>
<tbody>
<tr>
<td>low</td>
<td>low</td>
</tr>
<tr>
<td></td>
<td>medium</td>
</tr>
<tr>
<td></td>
<td>high</td>
</tr>
<tr>
<td>low</td>
<td>10%</td>
</tr>
<tr>
<td>medium</td>
<td>9%</td>
</tr>
<tr>
<td>high</td>
<td>6%</td>
</tr>
</tbody>
</table>

#### 4.

<table>
<thead>
<tr>
<th>Have a Summer Job</th>
<th>Gender</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>yes</td>
<td>boy</td>
<td>38%</td>
</tr>
<tr>
<td></td>
<td>girl</td>
<td>38%</td>
</tr>
<tr>
<td>no</td>
<td>boy</td>
<td>63%</td>
</tr>
<tr>
<td></td>
<td>girl</td>
<td>63%</td>
</tr>
</tbody>
</table>

#### 5.

<table>
<thead>
<tr>
<th>Play Sport</th>
<th>Grade</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>yes</td>
<td>6</td>
<td>7</td>
</tr>
<tr>
<td></td>
<td>43%</td>
<td>28%</td>
</tr>
<tr>
<td>no</td>
<td>58%</td>
<td>73%</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Play Sport</th>
<th>Grade</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>yes</td>
<td>6</td>
<td>7</td>
</tr>
<tr>
<td></td>
<td>40%</td>
<td>26%</td>
</tr>
<tr>
<td>no</td>
<td>29%</td>
<td>37%</td>
</tr>
</tbody>
</table>
Investigation 5 Check-Up

1. a.

<table>
<thead>
<tr>
<th>Gender</th>
<th>Candidate</th>
<th>Frequency</th>
</tr>
</thead>
<tbody>
<tr>
<td>girl</td>
<td>Julia</td>
<td>13</td>
</tr>
<tr>
<td></td>
<td>Edie</td>
<td>12</td>
</tr>
<tr>
<td></td>
<td>Bill</td>
<td>5</td>
</tr>
<tr>
<td></td>
<td>Frequency</td>
<td>30</td>
</tr>
<tr>
<td>boy</td>
<td>Julia</td>
<td>2</td>
</tr>
<tr>
<td></td>
<td>Edie</td>
<td>3</td>
</tr>
<tr>
<td></td>
<td>Bill</td>
<td>10</td>
</tr>
<tr>
<td></td>
<td>Frequency</td>
<td>15</td>
</tr>
</tbody>
</table>

The surveyed students were equally likely to vote for any of the three candidates, but Rama surveyed twice as many girls as boys.

b.

<table>
<thead>
<tr>
<th>Gender</th>
<th>Candidate</th>
<th>Julia</th>
<th>Edie</th>
<th>Bill</th>
</tr>
</thead>
<tbody>
<tr>
<td>girl</td>
<td>43%</td>
<td>40%</td>
<td>17%</td>
<td></td>
</tr>
<tr>
<td>boy</td>
<td>13%</td>
<td>20%</td>
<td>67%</td>
<td></td>
</tr>
</tbody>
</table>

Girls were fairly evenly split in their support of Julia and Edie, with fewer supporting Bill. Most boys support Bill, with fewer fairly evenly split between Julia and Edie.

c.

<table>
<thead>
<tr>
<th>Gender</th>
<th>Candidate</th>
<th>Julia</th>
<th>Edie</th>
<th>Bill</th>
</tr>
</thead>
<tbody>
<tr>
<td>girl</td>
<td>87%</td>
<td>80%</td>
<td>33%</td>
<td></td>
</tr>
<tr>
<td>boy</td>
<td>13%</td>
<td>20%</td>
<td>67%</td>
<td></td>
</tr>
</tbody>
</table>

The majority of each candidate’s support comes from students of that candidate’s gender.

d. Bill; Bill probably will receive most of the boys’ votes, while Julia and Edie probably will split the girls’ votes in half.
2. a. 

<table>
<thead>
<tr>
<th>Time Online</th>
<th>Frequency</th>
</tr>
</thead>
<tbody>
<tr>
<td>little</td>
<td>23</td>
</tr>
<tr>
<td>medium</td>
<td>37</td>
</tr>
<tr>
<td>a lot</td>
<td>20</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Good Grades?</th>
<th>yes</th>
<th>no</th>
</tr>
</thead>
<tbody>
<tr>
<td>little</td>
<td>16</td>
<td>7</td>
</tr>
<tr>
<td>medium</td>
<td>24</td>
<td>13</td>
</tr>
<tr>
<td>a lot</td>
<td>10</td>
<td>10</td>
</tr>
</tbody>
</table>

b. Most of the students surveyed say they spend a medium amount of time online, and more say they get good grades than do not.

c. 

<table>
<thead>
<tr>
<th>Good Grades?</th>
<th>Frequency</th>
</tr>
</thead>
<tbody>
<tr>
<td>yes</td>
<td>50</td>
</tr>
<tr>
<td>no</td>
<td>30</td>
</tr>
</tbody>
</table>

d. Most students with good grades spend a little or medium amount of time online, while most students without good grades spend a medium amount or a lot of time online.

e. Marta looked only at frequencies and not relative frequencies. There were more students with good grades surveyed, so the relative frequency of those students who spend a lot of time online is much less than for students without good grades.

f. Yes, according to the frequencies, there are almost twice as many students with good grades who spend a medium amount of time online, but the relative frequencies are close to being equal.