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## Common Core Investigations Teacher’s Guide

### Grade 6

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### Support for the Common Core Investigations

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### Answers for Additional Practices, Skill Practices, and Check-Ups ..... 53
Connected Mathematics (CMP) is a field-tested and research-validated program that focuses on a few big ideas at each grade level. Students explore these ideas in depth, thereby developing deep understanding of key ideas that they carry from one grade to the next. The sequencing of topics within a grade and from grade to grade, the result of lengthy field-testing and validation, helps to ensure the development of students’ deep mathematical understanding and strong problem-solving skills. By the end of grade 8, CMP students will have studied all of the content and skills in the Common Core State Standards for Mathematics (CCSSM) for middle grades (Grades 6-8). The focus on helping students develop deep mathematical understanding and strong problem solving skills aligns well to the intent of the Common Core State Standards for Mathematics, which articulates 3 to 5 areas of emphasis at each grade level from Kindergarten through Grade 8.

The sequence of content and skills in CMP varies in some instances from that in the CCSSM, so in collaboration with the CMP2 authors, Pearson has created a set of investigations for each grade level to further support and fully develop students’ understanding of the content standards of the CCSSM. The authors are confident that the CMP2 curriculum supplemented with the additional investigations at each grade level will address all of the content and skills of the CCSSM, but even more, will contribute significantly to advancing students’ mathematical proficiency as described in the Standards for Mathematical Practice of the CCSSM. Through the in-depth exploration of concepts, students become confident in solving a variety of problems with flexibility, skill, and insightfulness, and are able to communicate their reasoning and understanding in a variety of ways.

In this supplement, you will find support for all of the Common Core (CC) Investigations.

- The At-A-Glance page includes Teaching Notes and answers to all problems and exercises for the CC Investigation.
- The Additional Practice and Skill Practice pages can be reproduced for your students. These offer opportunities for students to reinforce the core concepts of the CC Investigation.
- Use the Check-Up to assess your students’ understanding of the concepts presented in the investigation.
- The answers for all of the ancillary pages are found at the back of this book.
- The reduced student pages are provided for your convenience as you read through the teaching support and plan for implementing each investigation.

In the Pacing Guide (pp. xii-xiii), we propose placement for teaching each CC Investigation. CC Investigations 1–4 require an understanding of operations with rational numbers, so they should be taught after Bits and Pieces III. CC Investigation 5 involves interpretation of data, so it should be used after the unit Data About Us.

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The following alignment of the Common Core State Standards for Mathematics to Pearson’s *Connected Mathematics 2* (CMP2) ©2009 program includes the supplemental investigations that complete the CMP2 program.

<table>
<thead>
<tr>
<th>COMMON CORE STATE STANDARDS GRADE 6</th>
<th>CMP2 UNIT</th>
<th>CONTENT</th>
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</thead>
<tbody>
<tr>
<td><strong>Ratios and Proportional Relationships</strong></td>
<td></td>
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</tr>
<tr>
<td>Understand ratio concepts and use ratio reasoning to solve problems.</td>
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</tr>
<tr>
<td>6.RP.1</td>
<td>Understand the concept of a ratio and use ratio language to describe a ratio relationship between two quantities.</td>
<td>Bits and Pieces I</td>
</tr>
<tr>
<td>6.RP.2</td>
<td>Understand the concept of a unit rate $a/b$ associated with a ratio $a:b$ with $b \neq 0$, and use rate language in the context of a ratio relationship.</td>
<td>CC Investigations</td>
</tr>
<tr>
<td>6.RP.3</td>
<td>Use ratio and rate reasoning to solve real-world and mathematical problems, e.g., by reasoning about tables of equivalent ratios, tape diagrams, double number line diagrams, or equations.</td>
<td>Bits and Pieces I, Shapes and Designs, How Likely Is It?</td>
</tr>
<tr>
<td>6.RP.3.a</td>
<td>Make tables of equivalent ratios relating quantities with whole number measurements, find missing values in the tables, and plot the pairs of values on the coordinate plane. Use tables to compare ratios.</td>
<td>Bits and Pieces I, CC Investigations</td>
</tr>
<tr>
<td>6.RP.3.b</td>
<td>Solve unit rate problems including those involving unit pricing and constant speed.</td>
<td>Bits and Pieces I, CC Investigations</td>
</tr>
<tr>
<td>6.RP.3.c</td>
<td>Find a percent of a quantity as a rate per 100 (e.g., 30% of a quantity means $30/100$ times the quantity); solve problems involving finding the whole, given a part and the percent.</td>
<td>Bits and Pieces III</td>
</tr>
<tr>
<td>6.RP.3.d</td>
<td>Use ratio reasoning to convert measurement units; manipulate and transform units appropriately when multiplying or dividing quantities.</td>
<td>CC Investigations</td>
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</table>
### COMMON CORE STATE STANDARDS GRADE 6

<table>
<thead>
<tr>
<th>CONTENT</th>
<th>CMP2 UNIT</th>
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</thead>
<tbody>
<tr>
<td><strong>The Number System</strong></td>
<td></td>
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</tbody>
</table>

#### 6.NS.1
Interpret and compute quotients of fractions, and solve word problems involving division of fractions by fractions, e.g., by using visual fraction models and equations to represent the problem.

- **Bits and Pieces II**
- **Inv. 4: Dividing with Fractions**

#### 6.NS.2
Fluently divide multi-digit numbers using the standard algorithm.

- **Bits and Pieces I**
- **Inv. 3: Moving Between Fractions and Decimals**
- **Inv. 3: The Decimal Divide**
- **Inv. 2: ACE 2, 29, 34, 35**

- **Bits and Pieces III**
- **Inv. 3: The Decimal Divide**

- **Shapes and Designs**

#### 6.NS.3
Fluently add, subtract, multiply, and divide multi-digit decimals using the standard algorithm for each operation.

- **Bits and Pieces III**
- **Inv. 1: Decimals–More or Less!**
- **Inv. 2: Decimal Times**
- **Inv. 3: The Decimal Divide**

#### 6.NS.4
Find the greatest common factor of two whole numbers less than or equal to 100 and the least common multiple of two whole numbers less than or equal to 12. Use the distributive property to express a sum of two whole numbers 1–100 with a common factor as a multiple of a sum of two whole numbers with no common factor.

- **Prime Time**
- **Inv. 2: Whole-Number Patterns and Relationships**
- **Inv. 3: Common Multiples and Common Factors**
- **CC Inv. 2: Number Properties and Algebraic Equations**

#### 6.NS.5
Understand that positive and negative numbers are used together to describe quantities having opposite directions or values (e.g., temperature above/below zero, elevation above/below sea level, credits/debits, positive/negative electric charge); use positive and negative numbers to represent quantities in real-world contexts, explaining the meaning of 0 in each situation.

- **Bits and Pieces II**
- **Inv. 2: ACE 51**

#### 6.NS.6
Understand a rational number as a point on the number line. Extend number line diagrams and coordinate axes familiar from previous grades to represent points on the line and in the plane with negative number coordinates.

- **Bits and Pieces I**
- **Inv. 1: Fundraising Fractions**
- **Inv. 2: Sharing and Comparing with Fractions**
- **Inv. 3: Moving Between Fractions and Decimals**
- **Inv. 4: Working with Percents**

- **Bits and Pieces II**
- **Inv. 1: Estimating with Fractions**
- **Inv. 2: Adding and Subtracting Fractions**
- **Inv. 3: Multiplying with Fractions**
- **Inv. 4: Dividing with Fractions**

- **Bits and Pieces III**
- **Inv. 1: Decimals–More or Less!**
- **Inv. 2: Decimal Times**
- **Inv. 3: The Decimal Divide**
- **Inv. 4: Using Percents**

#### 6.NS.6.a
Recognize opposite signs of numbers as indicating locations on opposite sides of 0 on the number line; recognize that the opposite of the opposite of a number is the number itself, e.g., $-(-3) = 3$, and that 0 is its own opposite.

- **Bits and Pieces II**
- **Inv. 2: ACE 51**
- **CC Inv. 3: Integers and the Coordinate Plane**

- **CC Investigations**

#### 6.NS.6.b
Understand signs of numbers in ordered pairs as indicating locations in quadrants of the coordinate plane; recognize that when two ordered pairs differ only by signs, the locations of the points are related by reflections across one or both axes.

- **CC Investigations**
- **CC Inv. 3: Integers and the Coordinate Plane**
<table>
<thead>
<tr>
<th>COMMON CORE STATE STANDARDS GRADE 6</th>
<th>CMP2 UNIT</th>
<th>CONTENT</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>6.NS.6.c</strong> Find and position integers and other rational numbers on a horizontal or vertical number line diagram; find and position pairs of integers and other rational numbers on a coordinate plane.</td>
<td>CC Investigations</td>
<td>CC Inv. 3: Integers and the Coordinate Plane</td>
</tr>
<tr>
<td><strong>6.NS.7</strong> Understand ordering and absolute value of rational numbers.</td>
<td>Bits and Pieces I</td>
<td>Inv. 1: Fundraising Fractions Inv. 2: Sharing and Comparing with Fractions Inv. 3: Moving Between Fractions and Decimals</td>
</tr>
<tr>
<td><strong>6.NS.7.a</strong> Interpret statements of inequality as statements about the relative position of two numbers on a number line diagram.</td>
<td>Bits and Pieces I</td>
<td>Inv. 1: Fundraising Fractions Inv. 2: Sharing and Comparing with Fractions Inv. 3: Moving Between Fractions and Decimals Inv. 4: Working with Percents</td>
</tr>
<tr>
<td><strong>6.NS.7.b</strong> Write, interpret, and explain statements of order for rational numbers in real-world contexts.</td>
<td>Bits and Pieces II</td>
<td>Inv. 2: ACE 51</td>
</tr>
<tr>
<td><strong>6.NS.7.c</strong> Understand the absolute value of a rational number as its distance from 0 on the number line; interpret absolute value as magnitude for a positive or negative quantity in a real-world situation.</td>
<td>CC Investigations</td>
<td>CC Inv. 3: Integers and the Coordinate Plane</td>
</tr>
<tr>
<td><strong>6.NS.7.d</strong> Distinguish comparisons of absolute value from statements about order.</td>
<td>CC Investigations</td>
<td>CC Inv. 3: Integers and the Coordinate Plane</td>
</tr>
<tr>
<td><strong>6.NS.8</strong> Solve real-world and mathematical problems by graphing points in all four quadrants of the coordinate plane. Include use of coordinates and absolute value to find distances between points with the same first coordinate or the same second coordinate.</td>
<td>Covering and Surrounding Data About Us CC Investigations</td>
<td>Inv. 2: Changing Area, Changing Perimeter Inv. 2: Using Graphs to Explore Data CC Inv. 3: Integers and the Coordinate Plane</td>
</tr>
</tbody>
</table>
### Expressions and Equations

**Apply and extend previous understandings of arithmetic to algebraic expressions.**

<table>
<thead>
<tr>
<th>Expression</th>
<th>Grade 6</th>
<th>CMP2 Unit</th>
<th>Content</th>
</tr>
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<tbody>
<tr>
<td><strong>6.EE.1</strong></td>
<td>Write and evaluate numerical expressions involving whole-number exponents.</td>
<td>Prime Time</td>
<td>Inv. 4: Factorizations: Searching for Factor Strings</td>
</tr>
<tr>
<td><strong>6.EE.2</strong></td>
<td>Write, read, and evaluate expressions in which letters stand for numbers.</td>
<td>Bits and Pieces II</td>
<td>Inv. 2: Adding and Subtracting Fractions, Inv. 3: Multiplying with Fractions, Inv. 4: Dividing with Fractions</td>
</tr>
<tr>
<td><strong>6.EE.2.a</strong></td>
<td>Write expressions that record operations with numbers and with letters standing for numbers.</td>
<td>Bits and Pieces II</td>
<td>Inv. 2: Adding and Subtracting Fractions, Inv. 3: Multiplying with Fractions, Inv. 4: Dividing with Fractions</td>
</tr>
<tr>
<td><strong>6.EE.2.b</strong></td>
<td>Identify parts of an expression using mathematical terms (sum, term, product, factor, quotient, coefficient); view one or more parts of an expression as a single entity.</td>
<td>Prime Time</td>
<td>Inv. 1: Factors and Products, Inv. 3: Common Multiples and Common Factors, Inv. 4: Factorizations: Searching for Factor Strings, Inv. 5: Putting It All Together</td>
</tr>
<tr>
<td><strong>6.EE.2.c</strong></td>
<td>Evaluate expressions at specific values of their variables. Include expressions that arise from formulas used in real-world problems. Perform arithmetic operations, including those involving whole number exponents, in the conventional order when there are no parentheses to specify a particular order (Order of Operations).</td>
<td>Covering and Surrounding</td>
<td>Inv. 1: Designing Bumper Cars, Inv. 2: Changing Area, Changing Perimeter, Inv. 3: Measuring Triangles, Inv. 4: Measuring Parallelograms, Inv. 5: Measuring Irregular Shapes and Circles</td>
</tr>
<tr>
<td><strong>6.EE.2</strong></td>
<td>Write, read, and evaluate expressions in which letters stand for numbers.</td>
<td>Bits and Pieces III</td>
<td>Inv. 1: Decimals–More or Less!, Inv. 2: Decimal Times, Inv. 3: The Decimal Divide, Inv. 4: Using Percents</td>
</tr>
<tr>
<td><strong>6.EE.2.a</strong></td>
<td>Write expressions that record operations with numbers and with letters standing for numbers.</td>
<td>Bits and Pieces III</td>
<td>Inv. 1: Decimals–More or Less!, Inv. 2: Decimal Times, Inv. 3: The Decimal Divide</td>
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<tr>
<td><strong>6.EE.2.b</strong></td>
<td>Identify parts of an expression using mathematical terms (sum, term, product, factor, quotient, coefficient); view one or more parts of an expression as a single entity.</td>
<td>CC Investigations</td>
<td>CC Inv. 2: Number Properties and Algebraic Equations</td>
</tr>
<tr>
<td><strong>6.EE.2.c</strong></td>
<td>Evaluate expressions at specific values of their variables. Include expressions that arise from formulas used in real-world problems. Perform arithmetic operations, including those involving whole number exponents, in the conventional order when there are no parentheses to specify a particular order (Order of Operations).</td>
<td>CC Investigations</td>
<td>CC Inv. 2: Number Properties and Algebraic Equations</td>
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<td>COMMON CORE STATE STANDARDS GRADE 6</td>
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<tr>
<td>6.EE.3 Apply the properties of operations to generate equivalent expressions.</td>
<td>CC Investigations</td>
<td>CC Inv. 2: Number Properties and Algebraic Equations</td>
<td></td>
</tr>
<tr>
<td>6.EE.4 Identify when two expressions are equivalent (i.e., when the two expressions name the same number regardless of which value is substituted into them).</td>
<td>CC Investigations</td>
<td>CC Inv. 2: Number Properties and Algebraic Equations</td>
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<tr>
<td><strong>Reason about and solve one-variable equations and inequalities.</strong></td>
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<tr>
<td>6.EE.5 Understand solving an equation or inequality as a process of answering a question: which values from a specified set, if any, make the equation or inequality true? Use substitution to determine whether a given number in a specified set makes an equation or inequality true.</td>
<td>Bits and Pieces II</td>
<td>Inv. 2: Adding and Subtracting Fractions</td>
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<td>Bits and Pieces III</td>
<td>Inv. 3: Multiplying with Fractions</td>
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<td>Shapes and Designs</td>
<td>Inv. 4: Dividing with Fractions</td>
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<td>Inv. 1: Decimals–More or Less!</td>
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<td>Inv. 2: Decimal Times</td>
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<td>Inv. 3: The Decimal Divide</td>
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<tr>
<td>6.EE.6 Use variables to represent numbers and write expressions when solving a real-world or mathematical problem; understand that a variable can represent an unknown number, or, depending on the purpose at hand, any number in a specified set.</td>
<td>Shapes and Designs</td>
<td>Inv. 3: ACE 30–33</td>
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<td>Inv. 4. Building Polygons</td>
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<td>Inv. 5: Making Irregular Shapes and Circles</td>
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<tr>
<td>6.EE.7 Solve real-world and mathematical problems by writing and solving equations of the form ( x + p = q ) and ( px = q ) for cases in which ( p, q ) and ( x ) are all nonnegative rational numbers.</td>
<td>Shapes and Designs</td>
<td>CC Inv. 2: Number Properties and Algebraic Equations</td>
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<td></td>
<td>Covering and Surrounding</td>
<td>Inv. 3: Polygon Properties and Tiling</td>
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<td></td>
<td>CC Investigations</td>
<td>Inv. 4. Building Polygons</td>
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<td></td>
<td>Inv. 5: Making Irregular Shapes and Circles</td>
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<tr>
<td>6.EE.8 Write an inequality of the form ( x &gt; c ) or ( x &lt; c ) to represent a constraint or condition in a real-world or mathematical problem. Recognize that inequalities of the form ( x &gt; c ) or ( x &lt; c ) have infinitely many solutions; represent solutions of such inequalities on number line diagrams.</td>
<td>CC Investigations</td>
<td>CC Inv. 3: Integers and the Coordinate Plane</td>
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<tr>
<td><strong>Represent and analyze quantitative relationships between dependent and independent variables.</strong></td>
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<tr>
<td>6.EE.9 Use variables to represent two quantities in a real-world problem that change in relationship to one another; write an equation to express one quantity, thought of as the dependent variable, in terms of the other quantity, thought of as the independent variable. Analyze the relationship between the dependent and independent variables using graphs and tables, and relate these to the equation.</td>
<td>Covering and Surrounding</td>
<td>Inv. 2: Changing Area, Changing Perimeter</td>
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<td>Inv. 2: Using Graphs to Explore Data</td>
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<td>CC Investigations</td>
<td>CC Inv. 2: Number Properties and Algebraic Equations</td>
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<td>COMMON CORE STATE STANDARDS GRADE 6</td>
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<tr>
<td>Geometry</td>
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<tr>
<td>Solve real-world and mathematical problems involving area, surface area, and volume.</td>
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<tr>
<td>6.G.1 Find the area of right triangles, other triangles, special quadrilaterals, and polygons by composing into rectangles or decomposing into triangles and other shapes; apply these techniques in the context of solving real-world and mathematical problems.</td>
<td>Covering and Surrounding</td>
<td>Inv. 1: Designing Bumper Cars, Inv. 2: Changing Area, Changing Perimeter, Inv. 3: Measuring Triangles, Inv. 4: Measuring Parallelograms, Inv. 5: Measuring Irregular Shapes and Circles</td>
<td></td>
</tr>
<tr>
<td>6.G.2 Find the volume of a right rectangular prism with fractional edge lengths by packing it with unit cubes of the appropriate unit fraction edge lengths, and show that the volume is the same as would be found by multiplying the edge lengths of the prism. Apply the formulas ( V = l \times w \times h ) and ( V = b \times h ) to find volumes of right rectangular prisms with fractional edge lengths in the context of solving real-world and mathematical problems.</td>
<td>CC Investigations</td>
<td>CC Inv. 4: Measurement</td>
<td></td>
</tr>
<tr>
<td>6.G.3 Draw polygons in the coordinate plane given coordinates for the vertices; use coordinates to find the length of a side joining points with the same first coordinate or the same second coordinate. Apply these techniques in the context of solving real-world and mathematical problems.</td>
<td>Shapes and Designs, CC Investigations</td>
<td>Inv. 2: ACE 39, CC Inv. 3: Integers and the Coordinate Plane</td>
<td></td>
</tr>
<tr>
<td>6.G.4 Represent three-dimensional figures using nets made up of rectangles and triangles, and use the nets to find the surface area of these figures. Apply these techniques in the context of solving real-world and mathematical problems.</td>
<td>Covering and Surrounding, CC Investigations</td>
<td>Inv. 3: ACE 39, CC Inv. 4: Measurement</td>
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### Develop understanding of statistical variability.

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<th>CMP2 Unit</th>
<th>Content</th>
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<tbody>
<tr>
<td>6.SP.1</td>
<td>Recognize a statistical question as one that anticipates variability in the data related to the question and accounts for it in the answers.</td>
<td></td>
<td>Inv. 1: Looking at Data</td>
<td>Inv. 2: Using Graphs to Explore Data Inv. 3: What Do We Mean by Mean? Unit Project</td>
</tr>
<tr>
<td>6.SP.2</td>
<td>Understand that a set of data collected to answer a statistical question has a distribution which can be described by its center, spread, and overall shape.</td>
<td></td>
<td>Inv. 1: Looking at Data Inv. 2: Using Graphs to Explore Data Inv. 3: What Do We Mean by Mean?</td>
<td></td>
</tr>
<tr>
<td>6.SP.3</td>
<td>Recognize that a measure of center for a numerical data set summarizes all of its values with a single number, while a measure of variation describes how its values vary with a single number.</td>
<td></td>
<td>Inv. 1: Looking at Data Inv. 2: Using Graphs to Explore Data Inv. 3: What Do We Mean by Mean?</td>
<td></td>
</tr>
</tbody>
</table>

### Summarize and describe distributions.

<table>
<thead>
<tr>
<th>Standard</th>
<th>Description</th>
<th>Data About Us</th>
<th>CMP2 Unit</th>
<th>Content</th>
</tr>
</thead>
<tbody>
<tr>
<td>6.SP.4</td>
<td>Display numerical data in plots on a number line, including dot plots, histograms, and box plots.</td>
<td>Data About Us</td>
<td>Inv. 1: Looking at Data Inv. 3: What Do We Mean by Mean? CC Inv. 5: Histograms and Box Plots</td>
<td></td>
</tr>
<tr>
<td>6.SP.5</td>
<td>Summarize numerical data sets in relation to their context, such as by:</td>
<td>Data About Us</td>
<td>Inv. 1: Looking at Data Inv. 2: Using Graphs to Explore Data Inv. 3: What Do We Mean by Mean?</td>
<td></td>
</tr>
<tr>
<td>6.SP.5.a</td>
<td>Reporting the number of observations.</td>
<td>How Likely Is It?</td>
<td>Inv. 1: A First Look at Chance Inv. 2: Experimental and Theoretical Probability Inv. 3: Making Decisions with Probability Inv. 4: Probability, Genetics, and Games</td>
<td></td>
</tr>
<tr>
<td>6.SP.5.b</td>
<td>Describing the nature of the attribute under investigation, including how it was measured and its units of measurement.</td>
<td>Data About Us</td>
<td>Inv. 1: Looking at Data Inv. 2: Using Graphs to Explore Data</td>
<td></td>
</tr>
<tr>
<td>6.SP.5.c</td>
<td>Giving quantitative measures of center (median and/or mean) and variability (interquartile range and/or mean absolute deviation), as well as describing any overall pattern and any striking deviations from the overall pattern with reference to the context in which the data were gathered.</td>
<td>Data About Us CC Inv. 5: Histograms and Box Plots</td>
<td></td>
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</tr>
<tr>
<td>6.SP.5.d</td>
<td>Relating the choice of measures of center and variability to the shape of the data distribution and the context in which the data were gathered.</td>
<td>Data About Us</td>
<td>Inv. 3: What Do We Mean by Mean?</td>
<td></td>
</tr>
</tbody>
</table>
This Pacing Guide offers suggestions as you look to implement the Grade 6 Common Core State Standards for Mathematics in the CMP2 classroom. The Chart shows placement recommendations for the Common Core Investigations provided in this supplement.

Investigations labeled as Review (R) offer timely practice of concepts from earlier grades, helping to activate students’ prior knowledge as they are introduced to new concepts that build on these concepts. Investigations labeled as Extending (*) offer students the opportunity to explore concepts in greater depth or to extend their study of concepts.

The suggested number of standard days for each unit is based on a 45-minute class period; a block period is assumed to be 90 minutes of instructional time. Common Core students entering Grade 6 are proficient in the concepts presented in Shapes and Designs, so you will be able to cover the concepts in this unit quickly; the concepts in How Likely Is It? align to Grade 7 standards and are addressed in the CMP2 Grade 7 Units, so you may choose to devote less time to these Investigations.

<table>
<thead>
<tr>
<th>Prime Time</th>
<th>Standard 22 days • Block 11 days</th>
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<tbody>
<tr>
<td>Inv. 1 Factors and Multiples</td>
<td>6.EE.2.b</td>
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<tr>
<td>Inv. 2 Whole Number Patterns and Relationships</td>
<td>6.NS.4</td>
</tr>
<tr>
<td>Inv. 3 Common Multiples and Common Factors</td>
<td>6.NS.4, 6.EE.2.b</td>
</tr>
<tr>
<td>Inv. 4 Factorizations: Searching for Factor Strings</td>
<td>6.EE.1, 6.EE.2.b</td>
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<tr>
<td>Inv. 5 Putting It All Together</td>
<td>6.EE.2.b</td>
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<tr>
<th>Bits and Pieces I</th>
<th>Standard 24 days • Block 12 days</th>
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<tbody>
<tr>
<td>Inv. 1 Fundraising Fractions</td>
<td>6.NS.6, 6.NS.7, 6.NS.7.a</td>
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<tr>
<td>Inv. 2 Sharing and Comparing With Fractions</td>
<td>6.NS.6, 6.NS.7, 6.NS.7.a</td>
</tr>
<tr>
<td>Inv. 3 Moving Between Fractions and Decimals</td>
<td>6.RP.3, 6.NS.2, 6.NS.6, 6.NS.7, 6.NS.7.a</td>
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<tr>
<td>Inv. 4 Working with Percents</td>
<td>6.RP.1, 6.RP.3, 6.RP.3.a, 6.RP.3.b, 6.NS.6, 6.NS.7.a</td>
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<table>
<thead>
<tr>
<th>Shapes and Designs</th>
<th>Standard 6 days • Block 3 days</th>
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<tbody>
<tr>
<td>Inv. 1 Bees and Polygons</td>
<td>Reviews 5.G.3, 5.G.4</td>
</tr>
<tr>
<td>Inv. 2 Polygons and Angles</td>
<td>Reviews 5.G.3, 5.G.4</td>
</tr>
<tr>
<td>Inv. 3 Polygons Properties and Tiling</td>
<td>Reviews 5.G.3, 5.G.4</td>
</tr>
<tr>
<td>Inv. 4 Building Polygons</td>
<td>Reviews 5.G.3, 5.G.4</td>
</tr>
</tbody>
</table>

**KEY**
- ✓ Core Content
- R Review
- * Extending
### Bits and Pieces II

<table>
<thead>
<tr>
<th>Activity</th>
<th>Standards</th>
<th>Duration</th>
<th>Notes</th>
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</thead>
<tbody>
<tr>
<td>Inv. 1 Estimating With Fractions</td>
<td>6.NS.6</td>
<td>22 days</td>
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</tr>
<tr>
<td>Inv. 2 Adding and Subtracting Fractions</td>
<td>6.NS.6, 6.EE.2.b, 6.EE.5</td>
<td>11 days</td>
<td>✓</td>
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<tr>
<td>Inv. 3 Multiplying With Fractions</td>
<td>6.NS.6, 6.NS.6.c, 6.EE.2.b, 6.EE.5</td>
<td>11 days</td>
<td>✓</td>
</tr>
<tr>
<td>Inv. 4 Dividing With Fractions</td>
<td>6.NS.1, 6.NS.6, 6.NS.6.c, 6.EE.2.b, 6.EE.5</td>
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### Covering and Surrounding

<table>
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<tr>
<th>Activity</th>
<th>Standards</th>
<th>Duration</th>
<th>Notes</th>
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<tbody>
<tr>
<td>Inv. 1 Designing Bumper Cars</td>
<td>6.G.1</td>
<td>27 days</td>
<td>✓</td>
</tr>
<tr>
<td>Inv. 2 Changing Area, Changing Perimeter</td>
<td>6.NS.8, 6.G.1</td>
<td>13 ½ days</td>
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<tr>
<td>Inv. 3 Measuring Triangles</td>
<td>6.G.1</td>
<td>13 ½ days</td>
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</tr>
<tr>
<td>Inv. 4 Measuring Parallelograms</td>
<td>6.G.1</td>
<td>13 ½ days</td>
<td>✓</td>
</tr>
<tr>
<td>Inv. 5 Measuring Irregular Shapes and Circles</td>
<td>6.G.1</td>
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</table>

### Bits and Pieces III

<table>
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<tr>
<th>Activity</th>
<th>Standards</th>
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<tbody>
<tr>
<td>Inv. 1 Decimals — More or Less</td>
<td>6.NS.3, 6.NS.6, 6.NS.6.c, 6.EE.2.b, 6.EE.5</td>
<td>45 days</td>
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<tr>
<td>Inv. 2 Decimal Times</td>
<td>6.NS.3, 6.NS.6, 6.EE.2.b, 6.EE.5</td>
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<tr>
<td>Inv. 3 The Decimal Divide</td>
<td>6.NS.2, 6.NS.3, 6.EE.2.b, 6.EE.5</td>
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<tr>
<td>Inv. 4 Using Percents</td>
<td>6.RP.3.c, 6.NS.6</td>
<td></td>
<td>✓</td>
</tr>
<tr>
<td>Inv. 5 More About Percents</td>
<td>6.RP.3.c</td>
<td></td>
<td>✓</td>
</tr>
<tr>
<td>CC Inv. 1 Ratios and Rates</td>
<td>6.RP.2, 6.RP.3.a, 6.RP.3.b, 6.RP.3.d</td>
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<tr>
<td>CC Inv. 2 Number Properties and Algebraic Equations</td>
<td>6.NS.4, 6.EE.2, 6.EE.2.a, 6.EE.2.b, 6.EE.2.c, 6.EE.3, 6.EE.4, 6.EE.6, 6.EE.7, 6.EE.9</td>
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<tr>
<td>CC Inv. 3 Integers and the Coordinate Plane</td>
<td>6.NS.6.a, 6.NS.6.b, 6.NS.6.c, 6.NS.7.c, 6.NS.7.d, 6.NS.8, 6.EE.8, 6.G.3</td>
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<td>6.G.2, 6.G.4</td>
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### How Likely Is It?

<table>
<thead>
<tr>
<th>Activity</th>
<th>Standards</th>
<th>Duration</th>
<th>Notes</th>
</tr>
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<tbody>
<tr>
<td>Inv. 1 A First Look at Chance</td>
<td>6.SP.5.a, Prepares for 7.SP.5</td>
<td>6 days</td>
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<tr>
<td>Inv. 2 Experimental and Theoretical Probability</td>
<td>6.SP.5.a, Prepares for 7.SP.5</td>
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<tr>
<td>Inv. 3 Making Decisions with Probability</td>
<td>6.SP.5.a, Prepares for 7.SP.1, 7.SP.5</td>
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<tr>
<td>Inv. 4 Probability, Genetics, and Games</td>
<td>6.SP.5.a, Prepares for 7.SP.1, 7.SP.5</td>
<td>6 days</td>
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### Data About Us

<table>
<thead>
<tr>
<th>Activity</th>
<th>Standards</th>
<th>Duration</th>
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</thead>
<tbody>
<tr>
<td>Inv. 1 Looking at Data</td>
<td>6.SP.1, 6.SP.2, 6.SP.3, 6.SP.4, 6.SP.5.a, 6.SP.5.b</td>
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<tr>
<td>Inv. 2 Using Graphs to Explore Data</td>
<td>6.NS.8, 6.SP.1, 6.SP.2, 6.SP.3, 6.SP.5.a, 6.SP.5.b</td>
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<tr>
<td>Inv. 3 What Do We Mean by Mean?</td>
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<tr>
<td>CC Inv. 5 Histograms and Box Blots</td>
<td>6.SP.4, 6.SP.5.c</td>
<td></td>
<td>✓</td>
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CC Investigation 1: Ratios and Rates

Mathematical Goals

- Understand the concept of a unit rate \( \frac{a}{b} \) associated with a ratio \( a : b \) with \( b \neq 0 \), and use rate language in the context of a ratio relationship.
- Make and tables of equivalent ratios, find missing values in the tables, and plot pairs of values on the coordinate plane.
- Solve unit rate problems.
- Use ratio reasoning to convert measurement units.

Teaching Notes

Proportional reasoning uses equivalent ratios to make comparisons. In this investigation, students solve problems that involve ratio—a comparison of two quantities. Success in solving ratio problems requires students to use the correct ratio and then to decide whether to multiply or divide.

Students also are introduced to rates, including unit rates. A rate is a comparison of two quantities with different units of measure, such as “8 packages for $29” or “50 miles per hour.” Have students use calculators as needed.

Students will use ratios to convert between customary and metric units of measurement. Success in converting units requires students to use the correct conversion factors and to decide whether to multiply or divide when making conversions.

Problem 1.1

Before Problem 1.1, have students read the definition of ratio and the examples. Explain the three ways of writing a ratio. Then ask:

- What do you think “the ratio of posters to minutes is 8 to 40” means? (Kai can make 8 posters in 40 minutes.)
- What is the ratio of minutes to posters? Explain. (40 to 8, because you have to match the units to the numbers.)

During Problem 1.1 ask: How can you find the number of posters per minute for each student? (First write the ratio of posters to minutes in fraction form. Then divide to find an equivalent fraction with a denominator of 1.)

Problem 1.2

During Problem 1.2 D, Part 1, ask: How can you write a ratio for each price tag? (number of collars to price or price to number of collars) Which way might be better in this situation? (price to number of collars)

After Problem 1.2 E, ask: Look at the price tags again. Can you eliminate some of the stores without calculating? (Pet Place and Dog Digs, because if you double each price, each is greater than $185.)
Problem 1.3

Before students begin Problem 1.3 B, ask: Where will you find the values that are represented on the vertical axis of the graph? (in the Weight column of the table) The horizontal axis? (in the Food column)

During Problem 1.3 D, ask: How will your table be similar to the Dogs table in the Problem, and how will it be different? (Both tables will have columns for Weight and Food, but the new table will not have a column for Name. The entries in the tables will be different.)

Problem 1.4

Before Problem 1.4, ask:
• How did you decide whether to use multiplication or division when writing a formula for converting inches to centimeters? (Since 1 in. = 2.54 cm, multiply inches by 2.54 to find the number of centimeters.)
• How can you convert centimeters to inches? (Since 2.54 cm = 1 in., divide centimeters by 2.54 to find inches: If $x = \text{length in cm}$ and $y = \text{length in inches}$, $x \div 2.54 = y$.)
• How could you calculate the number of inches in 1 centimeter? (Divide 1 cm by 2.54.) In 9 centimeters? (Divide 9 cm by 2.54.)

As students read through Problem 1.4 A, ask: Why is it necessary for Manuel to convert collar lengths? (To compare collar lengths, the measurements must be in the same units.)

During Problem 1.4 B, make sure students know how to convert millimeters to centimeters (10 mm = 1 cm). Ask: Why do you need to convert millimeters to centimeters? (If you convert millimeters to centimeters first, you can then use the conversion factor between centimeters and inches to convert millimeters to inches.)

Problem 1.5

After Problem 1.5 A, Part 1, ask: Once you know Ember’s mass in kilograms, how can you determine the number of cups of food that Ember should be fed each day? (Substitute the mass into the formula for food calories needed. Then divide the result by the number of calories in a cup.)

During Problem 1.5 B, Part 2, ask: Will your formula be different from the formula that was in the first table? (Yes; the formulas in the first table involved mass in kilograms. The new formula involves weight in pounds.)
Summarize
To summarize the lesson, ask:
- How can you find a unit rate? (Divide the numerator by the denominator.) Why is it important to understand unit rate? (to be able to compare and order unit rates, including unit prices)
- How can you use the information given in a table of ratios to fill in missing amounts? (Use given information to find the ratio shown by data pairs in the table, and then apply that to find any missing values.)
- How do you convert a measurement from customary to metric units? (Use the appropriate conversion factor with multiplication or division.)

Students in the CMP2 program will further study standards 6.RP.2, 6.RP.3.a, 6.RP.3.b, and 6.RP.3.d in the Grade 7 Unit Comparing and Scaling.

Assignment Guide for Investigation 1
Problem 1.1, Exercises 1–3, 19, 24–29
Problem 1.2, Exercises 4–18, 20–23, 30–31
Problem 1.3, Exercises 32–35
Problem 1.4, Exercises 36–38, 42–44, 46–49
Problem 1.5, Exercises 39–41, 45

Answers to Investigation 1
Problem 1.1
A. Selena: \(\frac{4}{80} = 4 \div 80 = 0.05\) posters per minute; Jason: \(\frac{4}{20} = 4 \div 20 = 0.2\) posters per minute; Kai: \(\frac{8}{40} = 8 \div 40 = 0.2\) posters per minute; Enrique: \(\frac{3}{30} = 3 \div 30 = 0.1\) posters per minute; Andre: \(\frac{3}{6} = 3 \div 6 = 0.5\) posters per minute
B. Andre; Selena
C. Selena’s new rate is \(\frac{8}{80} = 8 \div 80 = 0.1\); Selena and Enrique have the slowest rate.
D. 30 ÷ 6 = 5; 5 posters

Problem 1.2
A. 1. March 25: 26.43 mi/gal; March 29: 27.31 mi/gal; April 8: 25.54 mi/gal; April 14: 20.14 mi/gal
2. On April 14, because the unit rate for the miles per gallon was much lower than any other day.
B. About 14.2 gallons of gasoline. Divide the miles driven by the unit rate: 368 miles ÷ 25.92 miles per gallon = 14.2 gallons.
C. 1. days ≤ 6 = 38; days ≥ 7 = 12; 38 : 12
2. There are about 3 times as many days with 6 or fewer trips as there are days with 7 or more trips.
3. 1 or 2, 13 : 50; 3 or 4, 15 : 50; 5 or 6, 10 : 50; 7 or 8, 5 : 50; 9 or 10, 3 : 50; > 10, 4 : 50
4. \(\frac{12}{50} \times 30 = 7.2\); about 7 days
D. 1. The store with the lowest unit price will sell the least expensive collar.
2. Happy Pet, because it has the lowest unit price at $1.98 per collar.
3. $297; Multiply 150 × $1.98.
E. Just Dogs. He can buy 7 packages that will give him a total of 84 collars.
Problem 1.3

A. 1. Beauty, \( \frac{24}{2} \) or \( \frac{12}{1} \); Scruffy, \( \frac{48}{4} \) or \( \frac{12}{1} \);
Sport, \( \frac{36}{3} \) or \( \frac{12}{1} \)

2. All of the ratios are equivalent, so the dogs all are fed at the same rate based on their weights.

3. Fifi, 1 cup; Honey, 60 pounds

4. Use the ratio \( \frac{12}{1} \) to write and solve the proportion \( \frac{12}{1} = \frac{w}{f} \) where \( w \) represents the dog’s weight, and \( f \) represents the number of cups of food it gets.

B. 1.

Problem 1.4

A. 1. No; she can convert either measurement as long as she compares like units.

2. Divide 40 cm by 2.54 to convert to inches; 40 cm = 15.7 in., so the new collars are about 15.7 in. long.

B. Selena can convert mm to cm and then cm to in.

C. Dog Duds, 15.7 in. (or 40 cm); Cool Collars, 18 in. (or 45.7 cm); Pretty Pooches, 19.7 in. (or 50 cm)

Problem 1.5

A. 1. 15 kg; \( \frac{33}{2.2} \) = 15

2. About 3.6 cups; Ember needs 90(15) + 70, or 1,420 calories each day; 1,420 calories divided by 400 calories per cup is about 3.6 cups.

B. 1. Pomeranian \( \approx \) 275 calories;
Dachshund \( \approx \) 1,052 calories;
Labrador retriever \( \approx \) 3,056 calories

2. \( n = 90 \times \left( \frac{w}{2.2} \right) + 70 \)

3. Sample answer: A table shows the calories needed for specific weights. A formula can easily be used with any weight.

4. Pomeranian: about 0.7 cups;
Dachshund: about 2.6 cups;
Labrador retriever: about 7.6 cups
Exercises
1. The ratio of students who have a computer to those who do not.
2. 4 students
3. Do you walk to school?
4. a. 2 times
   b. 32 base hits
   c. about 6
5. June 25: 8 to 324; July 23: 10 to 410; August 8: 6 to 297; September 14: 9 to 502; October 6: 8 to 450; These ratios are already in order from least to greatest.
6. about 20 earphones
7. 60 females
8. 686 votes
9. 20 pages in 30 minutes
10. 132 miles in 4 hours
11. 273 students in 7 buses
12. 675 minutes or $\frac{111}{2}$ hours
13. $67.60$
14. 200 minutes or $3\frac{1}{3}$ hours
15. a. Reece
   b. Ava; 150 minutes or 2.5 hours
   c. Reece
   d. about 456 minutes or 7 hours 36 minutes
16. $\frac{4.75}{5\text{ lb}} = \$0.95 \text{ per pound}$
17. $\frac{3.72}{3\text{ kg}} = \$1.24 \text{ per kilogram}$
18. $\frac{1.05}{150 \text{ sheets}} = \$0.007 \text{ per sheet}$
19. a. By the Box, $\frac{7.29}{50 \text{ beads}}$;
   U.S. Crafts, $\frac{4.06}{38 \text{ beads}}$;
   Bella’s Beads, $\frac{8.76}{65 \text{ beads}}$;
   Jul’s Jewels, $\frac{6.76}{55 \text{ beads}}$;
   Crazy Crafts, $\frac{9.00}{78 \text{ beads}}$
   b. U.S. Crafts
   c. 8 containers; $\$32.48$
   d. Crazy Crafts; 156 beads
20. B
21. a. 40 ft
   b. 60 ft
22. a. 2.64 inches
   b. about 1.3 miles
23. a. 20 oz for $\$1.60$
   b. Olivia should buy two 16-oz cans and one 10-oz can to get exactly 42 oz.
24. $\frac{759}{22}$; 34.5 mi/gal
25. $\frac{3.01}{1.21}$; about $\$2.49/\text{lb}$
26. $\frac{25.92}{12}$; $\$2.16 \text{ per key chain}$
27. $\frac{72}{6}$; 12 calls per hour
28. $\frac{2220}{6}$; 370 Calories per serving
29. $\frac{270}{144}$; about $\$1.88 \text{ per American flag patch}$
30. a. 4-pack, $\$0.55/\text{bottle}$; 8-pack, $\$0.45/\text{bottle}$;
    24-pack, $\$0.29/\text{bottle}$
   b. 24-pack
   c. $\$0.25/\text{bottle}$
   d. three 8-packs
31. a. pages per day, days per page
   b. 24 pages per day means she can read 24 pages each day; 0.04 day per page means it takes her 0.04 day to read each page.
   c. 72 pages
   d. Yes; if she reads at the same rate, she will finish in time. 144 pages in 5 days is almost 29 pages per day, which is faster than 24 pages per day.
32. C, 15; D, 21; E, 9
33.  

34. Week 3, 42; Week 4, 5  

35. a.  

<table>
<thead>
<tr>
<th>Quarter Exchanges</th>
<th>Number of Nickels</th>
<th>Number of Quarters</th>
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</thead>
<tbody>
<tr>
<td>5</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td>15</td>
<td>3</td>
<td></td>
</tr>
<tr>
<td>25</td>
<td>5</td>
<td></td>
</tr>
<tr>
<td>35</td>
<td>7</td>
<td></td>
</tr>
<tr>
<td>50</td>
<td>10</td>
<td></td>
</tr>
</tbody>
</table>

b. \( \frac{5}{5} \) or \( \frac{1}{5} \)  
c. 72 quarters  

36. The DF 200  

37. Yes; \( \frac{1}{2} \) ft = 76.2 cm, and 70 cm < 76.2 cm  

38. A  

39. About 23.8 lb  

40. About 1.4 kg  

41. Yes; a 50-kilogram rock would weigh only about 18 pounds on the Moon.  

42. 3.75 cm/hr  

43. No; it was traveling at 62.1 mi/hr.  

44. 2,000 cm\(^3\)  

45. 10.14 newtons/cm\(^2\)  

46. About 44 mi/hr (cycling)  

47. C  

48. Yes; corresponding angles are congruent and corresponding sides are proportional, so the triangles are similar. These triangles are also congruent.  

49. Rodney’s; 1.82 ft\(^2\) or 0.17 m\(^2\)
1. Calvin is making trail mix to take on a hike. He has four different recipes he can use.

<table>
<thead>
<tr>
<th>Trail Mix Recipes</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Recipe</strong></td>
</tr>
<tr>
<td>A</td>
</tr>
<tr>
<td>B</td>
</tr>
<tr>
<td>C</td>
</tr>
<tr>
<td>D</td>
</tr>
</tbody>
</table>

a. Calvin’s sister doesn’t like too many nuts in her trail mix. She asks Calvin to use recipe B. Explain why Calvin’s sister might prefer a different recipe.

b. Calvin makes a special batch of trail mix using recipe B and adding more nuts. As he adds nuts, will he get to the nut-granola ratio of recipe A or recipe D first? Explain.

2. Rebecca is planning to visit Canada and drive from Ottawa to Halifax. She drives at an average speed of 65 miles per hour and won’t drive longer than 4 hours in a day.

a. Make a table to show how far Rebecca can drive in 1, 2, 3, and 4 hours. Then plot the pairs of values on a coordinate plane.

b. The table gives the distances between cities along Rebecca’s route. How many days will it take her to drive from Ottawa to Halifax? Explain. Use 1 km = 0.62 mi.

<table>
<thead>
<tr>
<th>Distances Between Canadian Cities</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Cities</strong></td>
</tr>
<tr>
<td>Ottawa to Montreal</td>
</tr>
<tr>
<td>Montreal to Quebec</td>
</tr>
<tr>
<td>Quebec to Halifax</td>
</tr>
</tbody>
</table>
Skill: Write Equivalent Fractions

Solve for \( x \) to make an equivalent fraction.

1. \( \frac{3}{4} = \frac{x}{8} \)  
2. \( \frac{1}{3} = \frac{x}{9} \)

3. \( \frac{1}{2} = \frac{7}{x} \)  
4. \( \frac{2}{5} = \frac{4}{x} \)

5. \( \frac{6}{2} = \frac{x}{1} \)  
6. \( \frac{5}{20} = \frac{x}{4} \)

7. \( \frac{36}{6} = \frac{x}{1} \)  
8. \( \frac{60}{12} = \frac{x}{8} \)

9. \( \frac{48}{100} = \frac{x}{25} \)  
10. \( \frac{3}{18} = \frac{1}{x} \)

Skill: Find the Unit Rate

For Exercises 11–20, write the fraction as a unit rate.

11. \( \frac{4}{8} \)  
12. \( \frac{5}{25} \)

13. \( \frac{36}{6} \)  
14. \( \frac{14}{21} \)

15. \( \frac{18}{30} \)  
16. \( \frac{28}{16} \)

17. \( \frac{56}{35} \)  
18. \( \frac{45}{108} \)

19. \( \frac{12}{42} \)  
20. \( \frac{39}{26} \)

21. Antoine paid $7.47 for 3 pounds of grapes. Write the cost of the grapes as a unit rate.

22. Carla’s family drove 420 miles in 8 hours. Write their average speed as a unit rate.
Check-Up

1. Zack, Tina, Ernie and Quinn are making bracelets to sell at a fair. The number of bracelets each student makes and the time it takes to make the bracelets are shown below. Each student wants to make 50 bracelets to sell at the fair.

<table>
<thead>
<tr>
<th>Student</th>
<th>Bracelets in</th>
<th>Time</th>
</tr>
</thead>
<tbody>
<tr>
<td>Zack</td>
<td>3</td>
<td>30 min</td>
</tr>
<tr>
<td>Tina</td>
<td>5</td>
<td>45 min</td>
</tr>
<tr>
<td>Ernie</td>
<td>4</td>
<td>28 min</td>
</tr>
<tr>
<td>Quinn</td>
<td>6</td>
<td>36 min</td>
</tr>
</tbody>
</table>

a. Determine which student is the fastest at making bracelets. Explain the method you used to find the answer.

b. Zack and Quinn work together for 1 hour. How many bracelets are they able to make?

2. After the first stage of an international bike race, cyclists post their average speed on a board. Some of the speeds are shown below.

<table>
<thead>
<tr>
<th>Cyclist</th>
<th>Home Country</th>
<th>Average Speed</th>
</tr>
</thead>
<tbody>
<tr>
<td>Marlon</td>
<td>France</td>
<td>44.8 km/h</td>
</tr>
<tr>
<td>Bernhard</td>
<td>Germany</td>
<td>44.0 km/h</td>
</tr>
<tr>
<td>Lance</td>
<td>United States</td>
<td>28.5 mi/h</td>
</tr>
<tr>
<td>Raul</td>
<td>United States</td>
<td>27.0 mi/h</td>
</tr>
</tbody>
</table>

a. Which of the cyclists took the least amount of time to complete the stage? Which one took the longest? Explain how you got your answers. Use 1 mi ≈ 1.6 km.

b. Raul averages 26 mi/h and 28.5 mi/h in the next two stages of the race. What is Raul’s average speed in km/h after the first three stages?
3. Your class is having a bake sale. You plan to make and sell one of three different types of cookies as shown in this chart.

<table>
<thead>
<tr>
<th>Type of Cookie</th>
<th>Cost Per Dozen Cookies to Make</th>
<th>Time to Make Per Dozen</th>
<th>Number of Cookies That Will Sell Per Hour</th>
</tr>
</thead>
<tbody>
<tr>
<td>Chocolate Chunk</td>
<td>$2.40</td>
<td>1 1/2 hours</td>
<td>18</td>
</tr>
<tr>
<td>Peanut Butter</td>
<td>$2.64</td>
<td>1 hour</td>
<td>12</td>
</tr>
<tr>
<td>Sugar</td>
<td>$2.88</td>
<td>3/4 hour</td>
<td>16</td>
</tr>
</tbody>
</table>

a. Find the unit cost to make each type of cookie.

b. You can sell the cookies for $0.75 each. Which type of cookie would generate the most profit per hour? Profit is the amount of sales minus the cost of the cookies sold.

c. You will have 6 hours to bake and sell your cookies. Which type of cookie should you make if you want to make the most profit? Explain how you found your answer.

d. One dozen oatmeal raisin cookies take 2 hours to make. One dozen cookies would cost $2.52, and the number of cookies you could sell in 1 hour would be 22. You still would have 6 hours to bake and sell the cookies, and you could sell them for $0.75 each. Should you sell oatmeal raisin cookies instead of any of the other types? Explain.
Some students volunteered to make posters for the animal shelter. The number of posters each student made and the time each student worked are shown in the table.

<table>
<thead>
<tr>
<th>Student Name</th>
<th>Number of Posters</th>
<th>Time in Minutes</th>
</tr>
</thead>
<tbody>
<tr>
<td>Selena</td>
<td>4</td>
<td>80</td>
</tr>
<tr>
<td>Jason</td>
<td>4</td>
<td>20</td>
</tr>
<tr>
<td>Kai</td>
<td>8</td>
<td>40</td>
</tr>
<tr>
<td>Enrique</td>
<td>3</td>
<td>30</td>
</tr>
<tr>
<td>Andre</td>
<td>3</td>
<td>6</td>
</tr>
</tbody>
</table>

A. How many posters per minute can each of the students make?

B. Which student makes the most posters per minute? The least?

C. If Selena makes 8 posters in 80 minutes, how does this change your answers to Parts A and B?

D. Matt joins the group, and it takes him 6 minutes to make each poster. How many posters can he make in 30 minutes?
A **ratio** is a comparison of two quantities. Fractions, decimals, and percents are ways to represent ratios. You can use the word “to,” a colon, or a fraction to write a ratio.

These statements contain ratios.
- For Kai, the ratio of posters to minutes is 8 to 40.
- For Kai, the ratio of posters to minutes is 8 : 40.
- For Kai, the ratio of posters to minutes is \(\frac{8}{40}\).

A **rate** is a ratio that compares quantities measured in different units.
- rate of production: 4 posters in 80 minutes
- rate of speed: 10 miles in 2 hours

A **unit rate** is a rate for which one of the numbers being compared is 1 unit.
- rate of production: 20 minutes per poster
- rate of pay: $15 for 1 hour

Selena, Jason, Kai, Enrique, and Andre will find using ratios very helpful in solving problems while they volunteer at the local animal shelter.
### Problem 1.2

**A.** Enrique drives the van for the animal shelter. He records the number of miles he drove and the amount of gasoline he purchased.

1. Copy and complete the table. Round to the nearest hundredth.

<table>
<thead>
<tr>
<th>Date</th>
<th>Miles Driven</th>
<th>Gallons of Gasoline Purchased</th>
<th>Miles/Gallon</th>
</tr>
</thead>
<tbody>
<tr>
<td>March 25</td>
<td>185</td>
<td>7.0</td>
<td></td>
</tr>
<tr>
<td>March 29</td>
<td>213</td>
<td>7.8</td>
<td></td>
</tr>
<tr>
<td>April 8</td>
<td>189</td>
<td>7.4</td>
<td></td>
</tr>
<tr>
<td>April 14</td>
<td>139</td>
<td>6.9</td>
<td></td>
</tr>
</tbody>
</table>

2. When Enrique notices that the miles per gallon is low, he drives to the mechanic. When do you think he went to the mechanic?

**B.** Enrique drives 368 miles at a rate of 25.92 miles per gallon. About how many gallons of gasoline did Enrique purchase?

**C.** The table shows the number of daily trips Enrique made for the 50 days he volunteered.

<table>
<thead>
<tr>
<th>Number of Daily Trips</th>
<th>Number of Trips</th>
<th>Number of Days</th>
</tr>
</thead>
<tbody>
<tr>
<td>1 or 2</td>
<td>13</td>
<td></td>
</tr>
<tr>
<td>3 or 4</td>
<td>15</td>
<td></td>
</tr>
<tr>
<td>5 or 6</td>
<td>10</td>
<td></td>
</tr>
<tr>
<td>7 or 8</td>
<td>5</td>
<td></td>
</tr>
<tr>
<td>9 or 10</td>
<td>3</td>
<td></td>
</tr>
<tr>
<td>&gt; 10</td>
<td>4</td>
<td></td>
</tr>
</tbody>
</table>

1. Use a ratio to compare the number of days Enrique made 6 or fewer trips to the number of days he made more than 6 trips.

2. What can Enrique determine from this ratio?

3. Write ratios that compare the number of days for each to the total number of days.

4. Over the next 30 days of volunteering, about how many days can Enrique expect to make 7 or more trips?
D. Andre is in charge of purchasing pet supplies for the animal shelter.  

1. How can you use unit price to find the store that sells the least expensive collar?  
2. From which store do you think Andre should buy collars?  
3. If Andre is asked to buy at least 150 collars, what is the least amount of money he needs?  

E. Andre is told to use $185 to buy the greatest number of collars possible. If he can purchase complete packages from only one store, where should he go to buy the collars?

Problem 1.3

Kai is responsible for feeding the animals at the shelter.  

A. He makes a table to record the amount of food each dog gets.  

<table>
<thead>
<tr>
<th>Dogs</th>
<th>Name</th>
<th>Weight (in pounds)</th>
<th>Food (in cups)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Beauty</td>
<td>24</td>
<td>2</td>
</tr>
<tr>
<td></td>
<td>Scruffy</td>
<td>48</td>
<td>4</td>
</tr>
<tr>
<td></td>
<td>Sport</td>
<td>36</td>
<td>3</td>
</tr>
<tr>
<td></td>
<td>Fifi</td>
<td>12</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Honey</td>
<td></td>
<td>5</td>
</tr>
</tbody>
</table>

1. Write ratios for Beauty, Scruffy, and Sport that compare the dog’s weight to the amount of food that the dog receives.  
2. Compare the ratios and explain what the comparison tells you about how the dogs are fed.  
3. Complete the table for Fifi and Honey.  
4. Explain how to use a ratio to find the amount of food any new dog at the shelter should receive.
B. Make a graph to show the feeding data.
   1. Plot the pairs of values in the table on a coordinate plane.
   2. Connect the points on the graph, and describe the shape that the data take. What does that shape tell you about the relationship between a dog’s weight and the amount of food it receives?
   3. What x-value corresponds to a y-value of 0 on the graph? Would this pair of values be the same for any ratio table? Explain your answer.

C. Kai uses another table to record feeding data for the shelter’s cats.

<table>
<thead>
<tr>
<th>Cats</th>
<th>Weight (in pounds)</th>
<th>Food (scoops)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Star</td>
<td>9</td>
<td>3</td>
</tr>
<tr>
<td>Frisky</td>
<td>6</td>
<td>2</td>
</tr>
<tr>
<td>Patch</td>
<td>12</td>
<td>4</td>
</tr>
<tr>
<td>Whiskers</td>
<td>3</td>
<td></td>
</tr>
<tr>
<td>Blackie</td>
<td>5</td>
<td></td>
</tr>
</tbody>
</table>

1. Write ratios for Star, Frisky, and Patch to compare the cat’s weight to the amount of food that the cat receives.
2. Complete the table for Whiskers and Blackie.
3. Compare the ratios for the dogs and the cats. How would a graph of the values for the cats differ from the graph you made for the dogs?

D. The shelter has a different food for older dogs. An older dog weighing 22 pounds gets 2 cups of the food, and an older dog weighing 55 pounds gets 5 cups of the food. Make a table showing the amounts of food to feed older dogs weighing 33 pounds, 44 pounds, and 77 pounds.
Comparing measurements is easy when they have the same unit. It’s not difficult to tell that a 10 1/2 -ounce can of juice contains less than a 12-ounce can. But when the units are different, comparing takes a bit more effort. You may need to change, or convert, one of the measurements so that both have the same unit.

**Getting Ready for Problem 1.4**
You can line up rulers to compare inches and centimeters.

<table>
<thead>
<tr>
<th>Inch Ruler</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
</tr>
<tr>
<td>2.54 cm</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Centimeter Ruler</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
</tr>
</tbody>
</table>

- What equation or formula could you use to convert inches to centimeters?
- How would you calculate the number of centimeters in 9 inches?
- What is a reasonable estimate for the number of inches in 1 centimeter?

**Problem 1.4**
The large dog collars that the shelter buys from Cool Collars measure 18 in. long. Selena is shopping for longer collars for the dogs. A company called Dog Duds sells large dog collars that measure 40 cm long. Selena wants to know which collar is longer.

A. Selena decided to begin by converting the collars’ dimensions.
   1. Does it matter if she converts inches to centimeters or centimeters to inches? Explain.
   2. Show how Selena can convert the length of the new collars to inches.

B. Selena found another store online, called Pretty Pooches, that sells large dog collars that measure 500 mm long. What unit conversions will Selena need to make to compare Pretty Pooches’ collars to the others?

C. List the collars in order of length from shortest to longest. Include the length of each store’s collars in one system of units.
The amount of food a dog needs depends on its weight and how active it is.

A. Craig just adopted Ember, a dog who weighs 33 pounds and is moderately active. Craig plans to use this table to help him decide how much to feed Ember.

<table>
<thead>
<tr>
<th>Level of Activity</th>
<th>Food Calories Needed Each Day</th>
</tr>
</thead>
<tbody>
<tr>
<td>Light activity</td>
<td>$60 \times m + 70$</td>
</tr>
<tr>
<td>Moderate activity</td>
<td>$90 \times m + 70$</td>
</tr>
<tr>
<td>Heavy activity</td>
<td></td>
</tr>
</tbody>
</table>

$m = \text{dog’s mass in kilograms}$

1. The mass 1 kilogram corresponds to a weight of about 2.2 pounds. What is Ember’s mass in kilograms?
2. About how many cups of food should Craig give Ember each day?

B. Craig is thinking of getting another dog and wants to see how much food he will need to buy regularly if he gets one of the dog breeds shown in the table. Craig plans on taking his dogs on daily walks, which is classified as moderate activity.

<table>
<thead>
<tr>
<th>Breed</th>
<th>Weight (lb)</th>
<th>Calories Needed Each Day</th>
</tr>
</thead>
<tbody>
<tr>
<td>Pomeranian</td>
<td>5</td>
<td></td>
</tr>
<tr>
<td>Dachshund</td>
<td>24</td>
<td></td>
</tr>
<tr>
<td>Labrador retriever</td>
<td>73</td>
<td></td>
</tr>
</tbody>
</table>

1. Copy and complete the table.

2. Write a formula for the number of calories a moderately-active adult dog needs daily if it weighs $w$ pounds.

3. Why might a formula be more useful than a table?

4. Use your formula to calculate the number of cups needed daily by each dog breed.
Exercises
For Exercises 1–3, use the table below that shows some facts about a class of 30 students.

<table>
<thead>
<tr>
<th>Class Facts</th>
<th>Yes</th>
<th>No</th>
</tr>
</thead>
<tbody>
<tr>
<td>Do you have a pet?</td>
<td>21</td>
<td>9</td>
</tr>
<tr>
<td>Do you have any brothers or sisters?</td>
<td>16</td>
<td>14</td>
</tr>
<tr>
<td>Do you have a computer?</td>
<td>22</td>
<td>8</td>
</tr>
<tr>
<td>Do you wear glasses?</td>
<td>10</td>
<td>20</td>
</tr>
<tr>
<td>Do you take music lessons?</td>
<td>9</td>
<td>21</td>
</tr>
<tr>
<td>Do you walk to school?</td>
<td>6</td>
<td>24</td>
</tr>
<tr>
<td>Have you traveled outside the United States?</td>
<td>5</td>
<td>25</td>
</tr>
<tr>
<td>Do you have a cell phone?</td>
<td>12</td>
<td>18</td>
</tr>
</tbody>
</table>

1. Which ratio is greater, the ratio of students who have a computer to those who do not, or the ratio of students who have a cell phone to those who do not?

2. If 12 students from this class were chosen at random, predict how many would wear glasses.

3. For which question did 4 out of 5 students in the class answer “No”?

4. Henry plays on a baseball team. Each of his 25 times at bat was recorded.

| Strikeouts | 10 |
| Base Hits  | 8  |
| Home Runs  | 4  |
| Walks      | 3  |

a. If this pattern continues and Henry gets to bat 5 times in the next game, predict the number of times he will strike out.

b. In 100 times at bat, how many base hits do you think Henry will get?

c. The team played in an 8-game tournament. If Henry had 48 times at bat and the pattern continued, about how many times was he walked?
The table below shows the results of an earphone company's product testing. Use this table to answer Exercises 5 and 6.

<table>
<thead>
<tr>
<th>Date</th>
<th>Number Tested</th>
<th>Number Defective</th>
</tr>
</thead>
<tbody>
<tr>
<td>June 25</td>
<td>324</td>
<td>8</td>
</tr>
<tr>
<td>July 23</td>
<td>410</td>
<td>10</td>
</tr>
<tr>
<td>August 8</td>
<td>297</td>
<td>6</td>
</tr>
<tr>
<td>September 14</td>
<td>502</td>
<td>9</td>
</tr>
<tr>
<td>October 6</td>
<td>450</td>
<td>8</td>
</tr>
</tbody>
</table>

5. Write a ratio for each date comparing the number of defective earphones to the number of earphones tested. Order the ratios from greatest to least.

6. On June 25, the company sent out an order of 800 earphones. How many of the earphones are expected to be defective?

7. In a local softball league, the ratio of males to females is 4 : 3. If there are 140 players in the league, how many are female?

8. In a recent election, the new mayor received three votes for every vote received by her opponent. The new mayor received 2,058 votes. How many votes did her opponent receive?

For Exercises 9–11, tell which rate is greater.

9. 18 pages in 36 minutes or 20 pages in 30 minutes

10. 132 miles in 4 hours or 62 miles in 2 hours

11. 273 students in 7 buses or 190 students in 5 buses
12. Miguel put 50 pieces of his puzzle together in 45 minutes. At that rate, how long will it take him in all to finish his 750-piece puzzle?

13. If your rate of pay is $10.40 per hour, how much will you earn if you work 6.5 hours?

14. The table shows how long it took a racecar in a 300-mile race to travel certain distances. If the car continued at the same rate of speed, how long did it take to complete the race?

<table>
<thead>
<tr>
<th>Distance</th>
<th>Time</th>
</tr>
</thead>
<tbody>
<tr>
<td>30 miles</td>
<td>20 min</td>
</tr>
<tr>
<td>60 miles</td>
<td>40 min</td>
</tr>
<tr>
<td>120 miles</td>
<td>80 min</td>
</tr>
</tbody>
</table>

15. To estimate the time it will take them to complete a 10-mile walk-a-thon, four friends record the distances they walk and their times.

<table>
<thead>
<tr>
<th>Name</th>
<th>Miles Walked</th>
<th>Time in Minutes</th>
</tr>
</thead>
<tbody>
<tr>
<td>Sean</td>
<td>3.6</td>
<td>80</td>
</tr>
<tr>
<td>Ava</td>
<td>2</td>
<td>30</td>
</tr>
<tr>
<td>Reece</td>
<td>5.5</td>
<td>132</td>
</tr>
<tr>
<td>Gail</td>
<td>0.6</td>
<td>18</td>
</tr>
</tbody>
</table>

a. Who walked faster, Reece or Gail?

b. At these rates, who will finish the walk-a-thon first, and how long will it take?

c. One of the friends can walk a mile in 24 minutes. Who is it?

d. If the four friends team up for a walking relay in which each team member walks 5 miles, how long will it take the team to finish the relay?

For Exercises 16–18, find each unit price. Show your work.

16. a 5-lb box of cat food for $4.75

17. a 3-kg bag of apples for $3.72

18. 150 sheets of paper for $1.05

Notes

______________________________

______________________________

______________________________

(10)10 Common Core Teacher’s Guide
19. Makayla found 5 different stores that sell the wood beads she wants for her craft project. The number of beads per container and the container price for each store is shown below. The beads can be purchased only as complete containers, and Makayla wants to buy the beads all from the same store.

<table>
<thead>
<tr>
<th>Store</th>
<th>Beads</th>
<th>Price</th>
</tr>
</thead>
<tbody>
<tr>
<td>By The Box</td>
<td>50</td>
<td>$7.29</td>
</tr>
<tr>
<td>U.S. Crafts</td>
<td>38</td>
<td>$4.06</td>
</tr>
<tr>
<td>Bella's</td>
<td>65</td>
<td>$8.76</td>
</tr>
<tr>
<td>Jul's Jewels</td>
<td>55</td>
<td>$6.76</td>
</tr>
<tr>
<td>Crazy Crafts</td>
<td>78</td>
<td>$9.00</td>
</tr>
</tbody>
</table>

a. Write a ratio comparing the container price to the number of beads in the container for each store.

b. At which store is the unit price of the beads the least?

c. Makayla needs 300 beads for her craft project. She wants to spend the least amount of money. How many containers of beads should she purchase and how much will they cost her?

d. If Makayla has $20 to spend, from which store can she buy the most beads, and how many beads can she get?

20. Multiple Choice Angela made some compost using 1 part coffee grounds, 6 parts food waste, 12 parts leaves, and 7 parts grass. She wants to adjust the mix so that it is \( \frac{1}{3} \) food waste. Which adjustment will work?

A. Add 2 more parts food waste and 2 more parts grass.
B. Add 4 more parts food waste.
C. Add 5 more parts food waste.
D. Add 4 more parts food waste and 4 more parts leaves.
21. To estimate the height of a pine tree near her school, Joy compared its shadow and the shadow of the school’s flagpole at the same time of day. She knows the flagpole is 30 feet tall.

   a. How tall is the pine tree?
   b. At the same time of day, the school building’s shadow is 42 feet long. How tall is the school building?

22. Each inch on a map represents 2,000 actual feet. This can be written as 1 in.: 2,000 ft.

   a. One mile is equivalent to 5,280 feet. About how many inches represent a mile on the map?
   b. Two cities on the map are 3.5 inches apart. What is the actual distance in miles between the two cities?

23. Below is a price list for canned peaches sold at two different stores.

<table>
<thead>
<tr>
<th>Sullivan’s Market</th>
<th>Dominick’s</th>
</tr>
</thead>
<tbody>
<tr>
<td>Can Size</td>
<td>Price</td>
</tr>
<tr>
<td>8 oz</td>
<td>$0.76</td>
</tr>
<tr>
<td>16 oz</td>
<td>$1.36</td>
</tr>
</tbody>
</table>

   a. Which size can is the best value?
   b. Olivia has a recipe for fruit salad. She needs 42 ounces of peaches. Which combination of cans should she buy? Explain.

For each Exercise 24–29, write each comparison as a rate. Then find the unit rate.

24. 759 miles per 22 gallons
25. $3.01 for 1.21 pounds of nectarines
26. $25.92 for 12 key chains
27. 72 telephone calls in 6 hours
28. 2,220 Calories in 6 servings
29. $270 for 144 American flag patches
30. a. Find the unit price for each size of packaging.
   b. Which size offers the best unit price?
   c. Find the new unit price for the 8-pack if it goes on sale for $1.99.
   d. What is the least expensive way to buy 24 bottles of water during the sale period?

31. Amanda must read 168 pages in her literature book in 7 days.
   a. What are the two unit rates that she might compute?
   b. Compute each unit rate and tell what it means.
   c. Amanda plans to read the same number of pages each day. How many pages should Amanda have read by the end of the third day?
   d. If she has read 144 pages by day 5, can she expect to finish in time? Explain.

For Exercises 32–33, use the table below that shows the sizes of some pens at the animal shelter. The ratio of each pen’s length to its area is the same.

<table>
<thead>
<tr>
<th>Pen Sizes</th>
</tr>
</thead>
<tbody>
<tr>
<td>Pen</td>
</tr>
<tr>
<td>------</td>
</tr>
<tr>
<td>A</td>
</tr>
<tr>
<td>B</td>
</tr>
<tr>
<td>C</td>
</tr>
<tr>
<td>D</td>
</tr>
<tr>
<td>E</td>
</tr>
</tbody>
</table>

32. Complete the table.

33. Plot the pairs of values in the table on a coordinate plane.
34. The table shows the money that Kyle makes mowing lawns. Kyle charges the same amount per lawn for each lawn he mows. Find the missing values.

<table>
<thead>
<tr>
<th>Week</th>
<th>Lawns Mowed</th>
<th>Total Income (dollars)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>6</td>
<td>84</td>
</tr>
<tr>
<td>2</td>
<td>2</td>
<td>28</td>
</tr>
<tr>
<td>3</td>
<td>3</td>
<td></td>
</tr>
<tr>
<td>4</td>
<td></td>
<td>70</td>
</tr>
</tbody>
</table>

35. Lori has a jar of nickels that she wants to exchange for quarters to use at an arcade.
   a. Make a table to show the numbers of quarters she can get in exchange for 5, 15, 25, 35, and 50 nickels.
   b. Write a ratio for each related pair of numbers in the table.
   c. Use the ratio to find how many quarters Lori can get in exchange for 360 nickels.

36. Louise is shopping online to buy a digital photo frame for her mom. Which frame gives her more display area per dollar?

<table>
<thead>
<tr>
<th>Frame</th>
<th>Size of Display</th>
<th>Price</th>
</tr>
</thead>
<tbody>
<tr>
<td>DF 200</td>
<td>8 in. × 10 in.</td>
<td>$59</td>
</tr>
<tr>
<td>X16</td>
<td>16 cm × 20 cm</td>
<td>$59</td>
</tr>
</tbody>
</table>

37. Ned wants to buy a stereo speaker that is 70 cm wide. Will the speaker fit in a space that is 2½ ft wide?

38. **Multiple Choice** Ashley wants to know if a stereo cabinet that is 1 m wide will fit in a space that is 40 in. wide. Which line of reasoning makes sense?
   A. The cabinet will just fit because the space is more than 40 \( \times \) 2.5, or 100 cm wide.
   B. The cabinet will not fit because it is 40 \( \times \) 2.54, or a little more than 100 cm wide.
   C. The cabinet will easily fit because it is about 1 \( \div \) 2.5, or much less than 1 cm wide.
   D. The cabinet will not fit because the space is 40 \( \div \) 2.54, or much less than 100 cm wide.
Use the paragraph below for Exercises 39–41.

Weight is the pull of gravity on an object. On Earth, a mass of 1 kilogram weighs about 2.2 pounds. On the Moon, gravity is weaker, so a 1-kilogram mass would weigh just about 0.36 pound. You can use this formula to convert between weight on the Moon and weight on Earth:

\[
\text{weight on Moon} = \text{weight on Earth} \div 6
\]

Note: weight on Earth in pounds ≈ 2.2 × mass in kilograms

39. An astronaut has a mass of 65 kilograms, so she weighs 143 pounds on Earth. What is her weight, in pounds, on the Moon?

40. A rock hammer weighs 8 ounces on the Moon. What is its mass, in kilograms?

41. Sam said he could easily pick up a 50-kilogram rock if he were on the Moon. Does Sam’s statement make sense?

42. Bamboo, the fastest growing plant in the world, can grow as much as 0.9 meters per day. Express the growth rate in centimeters per hour.

43. A car moved at a speed of 100 kilometers per hour on a highway with a speed limit of 65 miles per hour. Was the car exceeding the speed limit? Explain.

44. Josh learned that, in the metric system, 1 milliliter equals 1 cubic centimeter (cm³). How many cubic centimeters are in a 2-liter bottle of soda?

45. At sea level, Earth’s atmosphere exerts a pressure of 14.7 pounds per square inch on Earth’s surface. What is this pressure in newtons per square centimeter? Use 1 pound = 4.45 newtons.
Use the table below for Exercises 46 and 47.

<table>
<thead>
<tr>
<th>Event (Men's)</th>
<th>Winner</th>
<th>Time (s)</th>
</tr>
</thead>
<tbody>
<tr>
<td>100 m run</td>
<td>Usain Bolt</td>
<td>9.7</td>
</tr>
<tr>
<td>200 m run</td>
<td>Usain Bolt</td>
<td>19.3</td>
</tr>
<tr>
<td>100 m free-style swim</td>
<td>Alain Bernard</td>
<td>47.2</td>
</tr>
<tr>
<td>100 m butterfly swim</td>
<td>Michael Phelps</td>
<td>50.6</td>
</tr>
<tr>
<td>200 m cycling</td>
<td>Chris Hoy</td>
<td>10.2</td>
</tr>
<tr>
<td>400 m run</td>
<td>LaShawn Merritt</td>
<td>43.7</td>
</tr>
<tr>
<td>400 m hurdles</td>
<td>Angelo Taylor</td>
<td>47.2</td>
</tr>
</tbody>
</table>

46. What is the fastest speed, in miles per hour, of the results shown?

47. Multiple Choice Which of these athletes had an average speed of 44 miles per hour?
   A. Usain Bolt in the 100-meter run
   B. Usain Bolt in the 200-meter run
   C. Chris Hoy in the 200-meter cycling
   D. Michael Phelps in the 100-meter butterfly swim

48. Are the triangles below similar? Explain.

49. Rodney and his friend Emile exchanged sketches of their bedrooms. Whose bedroom has the greater area? How much greater?
CC Investigation 2: Number Properties and Algebraic Equations

Mathematical Goals

- Write, read, and evaluate expressions in which letters stand for numbers.
- Write expressions that record operations with numbers and with letters that stand for numbers.
- Identify the parts of an expression using mathematical terms.
- Evaluate expressions, including those arising from real-world formulas, for specific values of the variables.
- Apply the properties of operations to generate equivalent expressions.
- Identify when two expressions are equivalent.
- Use variables to write expressions to solve problems.
- Write equations using variables to represent two related quantities in a real-world problem.
- Analyze relationships between dependent and independent variables using graphs, tables, and equations.
- Use the distributive property to express the sum of two whole numbers with a common factor as a multiple of a sum of two whole numbers with no common factor.

Teaching Notes

In this investigation, students apply and extend previous understandings of arithmetic to algebraic expressions. Students will translate problem situations, such as “$15 per day,” or “some squash and twice as many gourds” into algebraic expressions that they can simplify for given values of the variables. Success in working with expressions requires students to understand that expressions in different forms can be equivalent, and that they can use the properties of operations to rewrite expressions in equivalent forms.

Students will reason about and solve both one- and two-variable equations. The properties of operations also will be applied here to solve equations. Students will learn to represent the relationship between dependent and independent variables using equations, tables, and graphs.

Vocabulary
- variable
- expression
- dependent variable
- independent variable
- commutative property
- associative property
- distributive property
- inverse operations

Materials
- calculator (optional)
Problem 2.1

Before Problem 2.1, have students read the definitions of variable and expression. Explain that it is important to translate words into correct operation symbols when writing an expression to describe a situation. Ask:

What is the variable in the expression $8n$? (n)

During Problem 2.1 A, ask:
- How can you find the amount of money collected for each number of visitors? (Multiply the number of visitors by 8.)
- What does each axis on your graph represent? (The x-axis will represent the number of visitors, and the y-axis will represent the amount of money collected.)
- Where does the graph cross the y-axis? (0)
- What does that point represent? (If there are no visitors, they collect $0.)

During Problem 2.1 B, ask: How can you find the profit each day? (Subtract the cost from the amount of money collected.)

During Problem 2.1 C, ask:
- What do negative values for $p$ represent? (losses)
- Where does the graph cross the y-axis? (0)
- What does that point represent? (If there are no visitors, there is a loss of $75.)

Problem 2.2

Before Problem 2.2, ask:
- Why would you need to write an expression with more than one variable? (to describe a situation where more than one amount is changing)
- What are some examples of situations where you would need two variables? (numbers of cats and dogs, horses and fields)

During Problem 2.2 A, ask:
- What expression represents the amount of money collected on Saturday? ($8n$)
- What expression represents the amount of money collected on Sunday? ($8m$)
- What are the independent variables in the expression? ($n$ and $m$)

During Problem 2.2 B, ask: How can you use your expression to find the profit for the weekend if you are given the total number of visitors? (Substitute 150 for $m + n$ and simplify.)

Problem 2.3

During Problem 2.3 A, ask: What expression represents the amount of money collected for tractor rides? ($3p$)

During Problem 2.3 B, ask: What operations will you use in your expression, and what do they represent? (multiplication to find the amount collected for each activity, addition to represent the total amount collected, subtraction to find the profit)
Problem 2.4

During Problem 2.4 A, ask: Which operation should you use to find the total time for a number of horses? (multiplication) Explain that to evaluate an expression, a student should substitute a number for a variable and simplify.

During Problem 2.4 B, ask: What is the formula for the area of a square? \((A = s^2)\)

Problem 2.5

During Problem 2.5 B, ask: How will the value of the expression you write in Part 1 change depending on which amount you decide to subtract? (The value will be either a positive or a negative number.)

Problem 2.6

Before Problem 2.6, explain that inverse operations undo each other, and that they are used to isolate a variable on one side of an equation to solve that equation. Write the equation \(x + 5 = 8\) on the board. Ask:

- What operation is used on the left side of the equation \(x + 5 = 8\)? (addition)
- What is the inverse operation for addition? (subtraction)
- How can you use subtraction to isolate the variable \(x\) on the left side of the equation? (Subtract 5 from each side of the equation.)

Problem 2.7

Before Problem 2.7, review the definition of perimeter. Then ask: What multiplication and addition formulas can you use to find the perimeter of a square with a side length \(s\)? \((P = 4s\ or\ P = s + s + s + s)\)

Summarize

To summarize the lesson, ask:

- When would you need to use an algebraic expression to represent a situation? (when there is an unknown quantity that can vary)
- How can you write an algebraic expression to solve a real-world problem? (Translate the words into an algebraic expression using one or more operations and variables. Then substitute values for the variables and simplify.)
- What information do you need to find the value of the expression \(5g + 7b\)? (the values of \(g\) and \(b\))
- How are dependent and independent variables related? (A dependent variable changes by a known amount as the independent variable is changed.)
- When might you use properties of operations? (when simplifying an expression to be able to use mental math)
Assignment Guide for Investigation 2

Problem 2.2, Exercises 5–6, 14–21, 26, 51–52, 89
Problem 2.3, Exercises 35–37
Problem 2.4, Exercises 7–8, 45–50
Problem 2.5, Exercises 54–64, 70
Problem 2.6, Exercises 27–34, 38, 65, 67–69, 71, 90–94
Problem 2.7, Exercises 72–78, 80–88, 95

Answers to Investigation 2

Problem 2.1
A. 1.

<table>
<thead>
<tr>
<th>Number of Visitors, ( n )</th>
<th>Amount of Money Collected, ( d )</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>$0</td>
</tr>
<tr>
<td>5</td>
<td>$40</td>
</tr>
<tr>
<td>10</td>
<td>$80</td>
</tr>
<tr>
<td>15</td>
<td>$120</td>
</tr>
<tr>
<td>20</td>
<td>$160</td>
</tr>
<tr>
<td>25</td>
<td>$200</td>
</tr>
<tr>
<td>30</td>
<td>$240</td>
</tr>
<tr>
<td>35</td>
<td>$280</td>
</tr>
<tr>
<td>40</td>
<td>$320</td>
</tr>
<tr>
<td>45</td>
<td>$360</td>
</tr>
<tr>
<td>50</td>
<td>$400</td>
</tr>
<tr>
<td>55</td>
<td>$440</td>
</tr>
<tr>
<td>60</td>
<td>$480</td>
</tr>
<tr>
<td>65</td>
<td>$520</td>
</tr>
<tr>
<td>70</td>
<td>$560</td>
</tr>
<tr>
<td>75</td>
<td>$600</td>
</tr>
<tr>
<td>80</td>
<td>$640</td>
</tr>
<tr>
<td>85</td>
<td>$680</td>
</tr>
<tr>
<td>90</td>
<td>$720</td>
</tr>
<tr>
<td>95</td>
<td>$760</td>
</tr>
<tr>
<td>100</td>
<td>$800</td>
</tr>
</tbody>
</table>

2. The graph is a straight line that passes through the origin.

B. 1. \( 8n - 75 \), where \( n \) represents the number of visitors

2. \( 8(80) - 75 = 640 - 75 = $565; \)
\( 8(105) - 75 = $765; 8(120) - 75 = $885 \)

C. 1.

<table>
<thead>
<tr>
<th>Number of Visitors, ( n )</th>
<th>Amount of Money Collected, ( d )</th>
<th>Profit, ( p )</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>$0</td>
<td>$-75</td>
</tr>
<tr>
<td>5</td>
<td>$40</td>
<td>$-35</td>
</tr>
<tr>
<td>10</td>
<td>$80</td>
<td>$5</td>
</tr>
<tr>
<td>15</td>
<td>$120</td>
<td>$45</td>
</tr>
<tr>
<td>20</td>
<td>$160</td>
<td>$85</td>
</tr>
<tr>
<td>25</td>
<td>$200</td>
<td>$125</td>
</tr>
<tr>
<td>30</td>
<td>$240</td>
<td>$165</td>
</tr>
<tr>
<td>35</td>
<td>$280</td>
<td>$205</td>
</tr>
<tr>
<td>40</td>
<td>$320</td>
<td>$245</td>
</tr>
<tr>
<td>45</td>
<td>$360</td>
<td>$285</td>
</tr>
<tr>
<td>50</td>
<td>$400</td>
<td>$325</td>
</tr>
<tr>
<td>55</td>
<td>$440</td>
<td>$365</td>
</tr>
<tr>
<td>60</td>
<td>$480</td>
<td>$405</td>
</tr>
<tr>
<td>65</td>
<td>$520</td>
<td>$445</td>
</tr>
<tr>
<td>70</td>
<td>$560</td>
<td>$485</td>
</tr>
<tr>
<td>75</td>
<td>$600</td>
<td>$525</td>
</tr>
<tr>
<td>80</td>
<td>$640</td>
<td>$565</td>
</tr>
<tr>
<td>85</td>
<td>$680</td>
<td>$605</td>
</tr>
<tr>
<td>90</td>
<td>$720</td>
<td>$645</td>
</tr>
<tr>
<td>95</td>
<td>$760</td>
<td>$685</td>
</tr>
<tr>
<td>100</td>
<td>$800</td>
<td>$725</td>
</tr>
</tbody>
</table>
This graph also is a straight line, but it intersects the y-axis at –75 instead of 0. The graphs are parallel.

Problem 2.2
A. \(8n + 8m\)
2. Yes; the expression \(8(n + m)\) is equivalent.
3. \(8n + 8m = 8(75) + 8(90) = 600 + 720 = 1,320\)

B. \(8n + 8m - 150\), or \(8(n + m) - 150\)
2. Yes, there is a profit of $1,050;
   \(8(n + m) - 150 = 8(150) - 150 = 1,200 - 150 = 1,050\)
3. 19 visitors; Set the money collected equal to the expenses and solve for total number of visitors: \(8(n + m) = 150; n + m = 150 ÷ 8 = 18.75\).

Problem 2.3
A. \(8n + 3t\), where \(n\) represents the number of visitors to the maze, and \(t\) represents the number of tractor rides
B. \(8n + 3t - 160\), where \(n\) represents the number of visitors to the maze, and \(t\) represents the number of tractor rides
C. \(8n + 3t - 160 = 8(70) + 3(40) - 160 = 560 + 120 - 160 = 520\)
\(8n + 3t - 160 = 8(90) + 3(65) - 160 = 720 + 195 - 160 = 755\)

Problem 2.4
A. \(30h\); The variable \(h\) represents the number of horses groomed.
2. \(30(5) = 150 \text{ min}, \text{ or } 2.5 \text{ hours}\)
3. \(150 - t\)
4. the amount of time \(t\) that Ben spent before lunch

B. \(20h\)
2. \(30h - 20h, \text{ or } 10h\)
3. \(10h = 10(4) = 40 \text{ min}; \text{ It takes Ben } 40 \text{ minutes longer than Emma to groom 4 horses.}\)

C. \(s^2 + s^2 + s^2, \text{ or } 3s^2\)
2. \(3s^2 = 3\left(\frac{1}{2}\right)^2 = 3\left(\frac{1}{4}\right) = \frac{3}{4} \text{ mi}^2\)

Problem 2.5
A. \(40f\)
2. \(30h + 40f\)
3. \(30h + 40f = 30(4) + 40(4) = 120 + 160 = 280 \text{ min}; \text{ It takes Ben } 280 \text{ minutes, or 4 hours 40 minutes, to groom 4 horses and mow 4 fields.}\)

B. \(30h - 40f\)
2. \(30h = 30(3) = 90 \text{ min}; 40f = 40(2) = 80 \text{ min}; \text{ Ben should choose to mow 2 fields since } 80 < 90, \text{ and he’ll be finished sooner.}\)
3. \(30h = 30(4) = 120 \text{ min}; 40f = 40(3) = 120 \text{ min}; \text{ Ben will spend 120 minutes doing either chore, so time should not be a factor in his choice.}\)
Problem 2.6
A. 1. \( g + 2 \)
   2. \( g + 2 = 16 \)
   3. By subtracting 2 from each side of the equation, we solve for \( g \), which represents the plant’s height last week.
   4. \( g + 2 = 16; g + 2 - 2 = 16 - 2; \)
   \( g = 14 \) in.
   5. twice the plant’s height last week
B. 1. \( m - 9 \)
   2. \( m - 9 = 25 \)
   3. Add 9; By adding 9 to each side of the equation, the variable \( m \) will be isolated on the left side, and the right side will be the value of \( m \).
   4. \( m - 9 = 25; m - 9 + 9 = 25 + 9; \)
   \( m = 34 \)
C. 1. \( 4s \)
   2. \( 4s = 168; \) division
   3. \( 4s ÷ 4 = 168 ÷ 4; s = 42 \) students
D. 1. \( p ÷ 5 \)
   2. \( p ÷ 5 = 32 \)
   3. \( p ÷ 5 = 32; 5(p ÷ 5) = 5(32); p = 160 \) sheets

Problem 2.7
A. 1. Both expressions are correct. Perimeter is the sum of the lengths of the sides, \( s + s + s + s \), which also equals \( 4s \).
   2. \( s + 2 + s + 2 + s + 2 + s + 2 \), or \( 4(s + 2) \), or \( 4s + 8 \)
B. 1. \( s + 2 + s + 2 + s + 2 = \)
   \( 6 + 2 + 6 + 2 + 6 + 2 + 6 + 2 = 32 \) ft;
   \( 10 + 2 + 10 + 2 + 10 + 2 + 10 + 2 = 48 \) ft;
   \( 13 + 2 + 13 + 2 + 13 + 2 + 13 + 2 = 60 \) ft
   2. \( 4(s + 2) = 4(6 + 2) = 32 \) ft; \( 4(10 + 2) = 48 \) ft;
   \( 4(13 + 2) = 60 \) ft
   3. The values are equal, so the expressions \( s + 2 + s + 2 + s + 2 + s + 2 \) and \( 4(s + 2) \) are equivalent.
C. Yes, the expressions are equivalent. According to the distributive property, \( 4(s + 2) = 4s + 8; \)
   \( 4s + 8 = 4(6) + 8 = 24 + 8 = 32 \) ft.

Exercises
1. \( j + 5 \)
2. \( a = s^2 \)
3. \( p = 1.35f + 12.5 \)
4. \( c + 0.15c = 12.95 \)
5. Super Locks: \( c = 3.975 + 6m; \) Fail Safe: \( c = 995 + 17.95m \)
6. Maggie: \( d = 1.250 - 70t; \) Ming: \( d = 800 - 40t \)
7. \( 12 = 1.4n + 0.2m \)
8. \( 300 = 15h - 30 \)
9. \( 0.49x \)
10. \( 0.306b \)
11. \( 10p \)
12. \( 20 - y \)
13. \( rt \)
14. 10
15. 60
16. 15
17. 6
18. \( 1\frac{1}{4} \)
19. \( \frac{1}{5} \)
20. \( \frac{1}{18} \)
21. 4
22. a length of time 24 hours longer than \( a \)
23. \( d \) fewer days than a year
24. each day for \( w \) weeks
25. \( m \) flowers divided among 55 tables
26. \( a. 3.95p \)
   \( b. 20 - 3.95p \)
   \( c. 5 \) packages
27. subtraction; \( a = 8 \)
28. addition; \( b = 12 \)
29. division; \( d = 3 \)
30. subtraction; \( t = 8 \)
31. multiplication; \( x = 10 \)
32. multiplication; \( n = 54 \)
33. addition; \( y = 42 \)
34. division; \( h = 12 \)
35. 31 people
36. 16 people
37. 9 packets of seeds
38. Becky should divide by 3 rather than subtract 3, because division is the inverse operation of multiplication.

39. \( 7n \)
40. \( n \div 6 \)
41. 84
42. 16
43. 15
44. 86
45. \( 80 + 20w; w \) represents the number of weeks she needs to pay off the violin.

46. 16 weeks
47. number of days for the rental
48. \( 45d + 25 \)
49. \( 50d \)
50. For \( d = 4 \), \( 45d + 25 = 45(4) + 25 = 180 + 25 = $205 \); \( 50d = 50(4) = $200 \); Grace and Tina should rent 2 single-seat canoes, because $200 < $205.

51. \( 20r + 8 \)
52. $68
53. \( 2g - 6 \)
54. a. \( 3x + 4y \)
   b. 300 plates and 200 forks
55. a. \( 375 \div z \)
   b. 5 packages
56. 94 tiles
57. the twelfth step

58. | Stage | 1 | 2 | 3 | 4 |
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Tiles</td>
<td>4</td>
<td>7</td>
<td>10</td>
<td>13</td>
</tr>
</tbody>
</table>

59. \( 4 + 3(s - 1) \), or \( 3s + 1 \)
60. 28 tiles
61. 33rd stage
62. \( y = \frac{x}{4} - 2 \)
63. Substitute 100 for \( x \) and simplify; \( y = 23 \).
64. Substitute 20 for \( y \) and simplify; \( x = 88 \).
65. B
66. a. \( 2w \)
   b. 80 meters
67. \( x = 6.9 \)
68. \( x = \frac{1}{10} \)
69. a. \( 5c - 20 = 20 \)
   b. \( c = $8 \)
70. a. \( 2.5m \)
   b. $155
71. Kelly is 8; Mike is 18.
72. associative property
73. distributive property
74. zero property
75. identity property
76. distributive property
77. commutative property
78. zero property
79. \(3n\); 21 ft, 30 ft, 45 ft

80. \(-4 + \frac{5}{2} + \frac{6}{5} + \frac{7}{2} + \frac{4}{5}\)

\[= -4 + \frac{5}{2} + \frac{7}{2} + \frac{6}{5} + \frac{4}{5}\]

commutative property;

\[= -4 + \left(\frac{5}{2} + \frac{7}{2}\right) + \left(\frac{6}{5} + \frac{4}{5}\right)\]

associative property;

\[-4 + \left(\frac{5}{2} + \frac{7}{2}\right) + \left(\frac{6}{5} + \frac{4}{5}\right) = -4 + \frac{12}{2} + \frac{10}{5}\]

\[= -4 + 6 + 2 = 4\]

81. \(5m + 6 + 3(m + 2) = 5m + 6 + 3m + 6\); distributive property; \(5m + 6 + 3m + 6 = 5m + 3m + 6 + 6\); commutative property;

\(5m + 3m + 6 + 6 = 8m + 12\)

82. \(-2\left(\frac{1}{2}k + \frac{1}{3}\right) + 6 + \frac{2}{3}\)

\[= -k - \frac{2}{3} + 6 + \frac{2}{3}\]; distributive property;

\[-k - \frac{2}{3} + \frac{2}{3} + 6\]; commutative property;

\[= -k + \left(-\frac{2}{3} + \frac{2}{3}\right) + 6\]; associative property;

\[= -k + 6\]; additive inverse property

83.

<table>
<thead>
<tr>
<th></th>
<th>2</th>
<th>5</th>
<th>10</th>
</tr>
</thead>
<tbody>
<tr>
<td>(3(x + 4))</td>
<td>18</td>
<td>27</td>
<td>42</td>
</tr>
<tr>
<td>(3x + 4)</td>
<td>10</td>
<td>19</td>
<td>34</td>
</tr>
<tr>
<td>(3x + 12)</td>
<td>18</td>
<td>27</td>
<td>42</td>
</tr>
<tr>
<td>(2(x + 6) + x)</td>
<td>18</td>
<td>27</td>
<td>42</td>
</tr>
<tr>
<td>(5x + (2x + 12))</td>
<td>26</td>
<td>47</td>
<td>82</td>
</tr>
</tbody>
</table>

\(3(x + 4); 3x + 12; \text{ and } 2(x + 6) + x\)

84. D

85. b

86. c

87. d

88. a

89. a. \(e = 9h\)

b. \(e\) is the dependent variable, and \(h\) is the independent variable.

c. 72; 54; 63; 72; 36; 45

90. \(d = 18t\)

91. \(y + 8 = x, \text{ or } y = x - 8\)

92. \(c = 2\pi r\)

93. \(w = z ÷ 1,000\)

94. B

95. a. 0.75; \(c \cdot 0.75\)

b. \(21 + 7p; 3(7) + 7p\)

c. \(7a + 14 + 3; 7a + 17\)
1. Cleave wants to go mountain biking. He found advertisements for two bicycle rental shops.

![Bob's Bike Rentals](attachment://Bob's_Bike_Rentals.png)

**Bob's Bike Rentals**

Rental Fee for one bike: $9 plus $3 per hour

![Cycle Center Bike Rentals](attachment://Cycle_Center_Bike_Rentals.png)

**Cycle Center Bike Rentals**

Rental Fee for one bike: $5 per hour

a. What other information would you need to find the total cost of renting a bike at either shop?

b. What variable will you use to represent the missing information?

c. Write expressions to find the costs of renting a bicycle at Bob’s Bike Rentals and Cycle Center Bike Rentals.

d. Make a table to show the costs of renting a bicycle at each of the shops for 1, 2, 3, 4, 5, and 6 hours.

e. Cleave thinks his mountain-biking trip will take between 4 and 6 hours. From which shop should he rent his bicycle? Explain.

2. A farmer sells asparagus for $3.75 per bundle, and broccoli for $2.25 per bundle. Elizabeth and Reece want to buy 6 bundles of each.

a. Elizabeth says she can find the total cost by multiplying the number of bundles of each food by its price, and then adding the costs. Write and evaluate an expression Elizabeth would use to find the total cost.

b. Reece says he can find the total cost by first adding the costs of one bundle of each food and then multiplying that sum by 6. Write and evaluate the expression Reece would use to find the total cost.

c. Compare the costs using Elizabeth’s and Reece’s methods. What does that tell you about the expressions you wrote?

d. Which method was easier for you? Explain.
Skill: Write and Evaluate Expressions

For Exercises 1–4, write an algebraic expression.

1. Misty has 36 pairs of shoes arranged on a number of shelves.

2. Martin buys 3 pounds of apples at a price per pound.

3. Twenty-four of the available seats in a movie theater are empty.

4. Paige is 6 inches taller than her brother.

5. Evaluate $4x - 7$ for $x = 9$.

6. Evaluate $54 - 3x$ for $x = 8$.

7. Evaluate $\frac{x}{4} \div 8$ for $x = 64$.

Skill: Work Backward

Music Universe is an online website for downloading music. Austin spent $26.30 during his first month as a club member.

8. How many songs did Austin download during the first month? Show your work.

9. Austin downloaded the same number of songs each month during the next 2 months. If he spent a total of $10.20, how many songs did he download each month? Explain.

10. Music Universe offers members 10 free downloads for referring a friend to the club. Austin referred his friend Michael to the club during June. Austin’s bill at the end of the month was $6.60. How many songs did he download during June? Explain.

11. Austin has decided on a monthly budget of $6.00 for his downloaded music. So far this month, he has spent $4.20. How many more songs can Austin download with the amount he has left in his budget? Show your work.
Check-Up

1. It takes Carlos 12 minutes to complete one section in his History workbook or complete one page in his Science book.
   a. Write an expression using a variable that shows how long it takes Carlos to complete some sections in his History workbook. Explain what your variable represents.

   b. Evaluate your expression to find how long it would take Carlos to complete 8 sections in his History workbook.

   c. Write an expression using variables that shows how long it takes Carlos to complete some History sections and complete some pages in his Science book.

   d. Write a different expression that also shows how long it takes Carlos to complete some History sections and some pages in his Science book. Explain how you know these expressions are equivalent.

   e. For homework one night, Carlos has 7 sections of History and 11 pages of Science to complete. Evaluate one of your expressions to find how long it will take Carlos to finish his homework.

2. Antoine has 14 quarters and 17 dimes. Ella has 17 quarters and some dimes.
   a. Write an expression using a variable to show how many coins Ella has. Evaluate the expression for 2, 5, 10, and 25 dimes.

   b. If Ella has the same number of coins as Antoine, write an equation setting their numbers of quarters and dimes equal. Explain how you can use a property to find the number of dimes Ella has.
3. Marcy is the summer camp director. She is ordering supplies that will be needed for the arts and crafts classes this summer. The table shows some of the items she will be ordering and the cost for each package.

<table>
<thead>
<tr>
<th>Item</th>
<th>Cost Per Package</th>
</tr>
</thead>
<tbody>
<tr>
<td>Acrylic Paint</td>
<td>$4.00</td>
</tr>
<tr>
<td>Beads</td>
<td>$2.50</td>
</tr>
<tr>
<td>Craft sticks</td>
<td>$3.00</td>
</tr>
</tbody>
</table>

a. The campers are divided into groups. Each group will need 4 packages of acrylic paints and 9 packages of beads. Write an expression to represent the total cost of the acrylic paints and beads that each group will need.

b. Each group also will need 6 packs of craft sticks. Write and evaluate an expression to find the total cost of one group’s supplies. Show your work.

c. Marcy decides to order more supplies. She orders $20 of acrylic paints and $15 worth of beads. Write and evaluate an expression to find the total number of packages Marcy ordered.

d. Marcy needs to order 3 packages of modeling clay for each group of campers. A package of modeling clay costs twice as much as a package of beads. Write an expression to represent the cost of the modeling clay for one group of campers.

e. If Marcy’s total budget for arts and crafts supplies is $400, will she be able to buy a full set of supplies for 6 groups of campers? Explain.
During the fall, Ben’s family operates a corn maze on their farm. They charge $8 per visitor. The total amount of money they collect is the number of visitors times $8. If $n$ represents the number of visitors, then the total amount of money collected is $8n$.

A variable is a letter or symbol that represents a quantity that can change. $8n$ is an expression that represents a quantity. In this situation, $8n$ is the total amount of money collected.

You can use variables and expressions to solve problems.

**Problem 2.1**

A. The family wanted an estimate of how much money they might receive from maze visitors on any given day. They decided to make a table that would display this amount for several different numbers of visitors for one day.

1. Copy and complete the table for numbers of visitors in increments of 5, starting with 0 and ending with 100.

<table>
<thead>
<tr>
<th>Number of Visitors, $n$</th>
<th>Amount of Money Collected, $d$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0</td>
</tr>
</tbody>
</table>

2. Sketch a graph of the number of visitors and the total amount of money collected.

3. Describe the shape of the graph.

B. Ben estimates that the cost of maintaining and advertising the maze is $75 per day.

1. Write an expression that represents the amount of profit that the family expects to make each day the maze is open.

2. Calculate the profit for 80 visitors, 105 visitors, and 120 visitors.

C. 1. Add a third column to the table in Part A that represents the profit, $p$.

2. Graph the number of visitors and profit on the same graph as in Part A.

3. Compare the shape of this graph to the one in Part A.
The total amount of money collected also is a variable. It depends on the number of visitors for any given day. We call the total amount of money collected the **dependent variable** and the number of visitors the **independent variable**.

### Problem 2.2

The corn maze is open on weekends. Let \( n \) represent the number of visitors on Saturday, and let \( m \) represent the number of visitors on Sunday.

A. 1. Write an expression that represents the total amount of money collected for both days.

   2. Ben claims there is more than one way to write this expression. Do you agree? Explain.

   3. What is the total amount of money collected for the weekend if there were 75 visitors on Saturday and 90 visitors on Sunday?

B. 1. Write an expression for the total profit for both days if the expenses are $75 per day.

   2. Is there a profit for a total of 150 visitors on both days? Explain.

   3. What is the least number of visitors needed on a weekend to break even? This is when the total revenue, the money taken in, equals the total expenses. Explain how you found your answer.

### Problem 2.3

The family charges $3 per person for tractor rides around the farm.

A. Write an expression that represents the total amount of money collected in one day from visitors to the maze and tractor rides. Explain what your variables represent.

B. If the combined expenses for the maze and tractor rides are $160 per day, write an expression for the profit for one day. Explain what your variables represent.

C. Calculate the profit for each.

1. 70 maze visitors and 40 tractor rides

2. 90 maze visitors and 65 tractor rides
Ben and his older sister, Emma, help out on their family’s farm by grooming horses and mowing the fields.

A. It takes Ben 30 minutes to groom each horse on the farm.
   1. Write an algebraic expression that shows how long it takes Ben to groom some horses. Explain what the variable in your expression represents.
   2. How long does it take Ben to groom 5 horses?
   3. Ben spends $t$ minutes grooming horses before lunch. Write an algebraic expression to show how much time Ben will need after lunch to finish grooming the 5 horses.
   4. What information do you need to be able to evaluate the expression?

B. It takes Ben’s older sister, Emma, 20 minutes to groom each horse on the farm.
   1. Write an expression to show how long it takes Emma to groom $h$ horses.
   2. Write an expression to show how much longer it takes Ben to groom $h$ horses than it takes Emma.
   3. Evaluate your expression for $h = 4$. Explain what the value means.

C. Three of the fields on the farm are squares with the same area.
   1. Write an expression to show the total area of the 3 fields in terms of their side length $s$.
   2. Evaluate your expression to find the total area of the fields if they each measure $\frac{1}{2}$ mile on a side.

A. It takes Ben 40 minutes to mow each field on the farm.
   1. Write an algebraic expression to represent the time that Ben spends mowing $f$ fields.
   2. Write an expression that shows how long it takes for Ben to groom $h$ horses and mow $f$ fields.
   3. Evaluate your expression for 4 horses and 4 fields. Explain what the value means.
B. Ben’s mother tells him that he needs either to groom some horses or mow some fields before he can go to a friend’s house.

1. Write an expression that shows how much longer it will take Ben to do one chore than the other.

2. Ben’s mother gives him the option of grooming 3 horses or mowing 2 fields. Which should Ben choose? Explain your answer.

3. Suppose Ben’s option is to groom 4 horses or mow 3 fields. Which should Ben choose? Explain your answer.

Problem 2.6

A. Emma knows that her plant is growing about 2 inches each week.

1. If \( g \) represents last week’s height of the plant in inches, write an expression for the height of the plant this week.

2. Today the plant measures 16 inches in height. Set your expression equal to 16.

3. How does subtracting 2 find the height of the plant last week?

4. How tall was the plant last week?

5. What would the expression \( 2g \) mean?

B. Ben just bought a rake for $9. He forgot how much money he had when he entered the hardware store.

1. If \( m \) represents the amount of money he had before he bought the rake, write an expression that represents the amount of money he has now.

2. He counts his money and finds that he has $25 left after he bought the rake. Set your expression equal to 25.

3. Ben wants to find the value of \( m \). He does not know whether he should add or subtract 9. Determine which operation is correct and explain your decision.

4. How much money did Ben have before he bought the rake?

C. Each student pays $4 to enter the school dance.

1. If \( s \) represents the number of students attending the dance, write an expression for the amount of money collected for the dance.

2. The money collected totals $168. Set your expression equal to 168. Which operation do you need to solve for \( s \)?

3. How many students came to the dance?
D. Christopher distributes sheets of paper to the class for a project. He gives each student 5 sheets. He wants to know how many sheets of paper he distributed.

1. If \( p \) represents the total number of sheets of paper, write an expression that represents the number of students in the class.

2. There are 32 students in the class. Set your expression equal to 32.

3. How many sheets of paper did Christopher distribute?

You can use the properties of operations described below to generate equivalent expressions.

**Commutative Property:**
Changing the order does not change the sum or product.

\[ 3 + 7 = 7 + 3 \quad 4 \times 5 = 5 \times 4 \]

**Associative Property:**
Changing the grouping does not change the sum or product.

\[ 4 + (7 + 9) = (4 + 7) + 9 \quad (6 \times 2) \times 8 = 6 \times (2 \times 8) \]

**Distributive Property:**
The product of a number times a sum is equal to the sum of the products of that number and each addend.

\[ 5 \times (6 + 11) = (5 \times 6) + (5 \times 11) \]

**Problem 2.7**

Ahmad and Shada’s aunt keeps some square tomato gardens on her farm. This summer, rabbits have been eating the tomatoes. Ahmed learned that marigolds keep rabbits away from tomato plants. He decides to help his aunt by planting a 1-foot border of marigolds around each tomato garden.

A. 1. Ahmad writes an expression for the perimeter of a garden as \( s + s + s + s \). Shada writes the expression \( 4s \) to represent the perimeter. Whose expression is correct? Explain how you know.

2. Write an expression to represent the perimeter of a garden after a 1-foot border of marigolds is added.
B. Three of the tomato gardens have side lengths of 6 feet, 10 feet, and 13 feet.

1. Ahmed uses the expression \( s + 2 + s + 2 + s + 2 + s + 2 \) to find the perimeter after the border of marigolds is added. Use this expression to find the perimeter of each size garden.

2. Shada uses the expression \( 4(s + 2) \) to find the perimeter after the border of marigolds is added. Use this expression to find the perimeter of each size garden.

3. What do you notice about the perimeter of each garden found using the different expressions? Explain what that tells you about the expressions.

C. Their uncle says that the outside perimeter of any garden also could be found using the expression \( 4s + 8 \). Is this expression equivalent to those written by Ahmed and Shada? Explain your reasoning using the garden with side lengths of 6 feet.

Exercises

In Exercises 1–4, write an algebraic expression or equation for each.

1. five years older than Jamal’s age
2. The area is the length of a side squared.
3. The price is $1.35 per flower plus $12.50 for the vase.
4. The cost of the meal plus the 15% tip came to $12.95.
5. Super Locks charges $3,975 to install a security system and $6.00 per month to monitor the system and respond to alerts. Fail Safe charges $995 to install and $17.95 per month. Write an equation for each company relating its total cost to the number of months.
6. Maggie lives 1,250 meters from school. Ming lives 800 meters from school. Maggie walks at an average speed of 70 meters per minute, while Ming walks at an average speed of 40 meters per minute. Write equations that show Maggie and Ming’s distances from school \( t \) minutes after they leave their homes.
7. Chris has $12 to spend on prints from his digital camera. He wants one 5-in. × 7-in. print and some 4-in. × 6-in. prints. Write an equation to find how many prints he can order if the price of each 5-in. × 7-in. print is $1.40 and the 4-in. × 6-in. prints are $.20 each.
8. Jamal has a tutoring job. He charges $15 per hour. Next month, he expects his expenses to be $30. Write an equation to find the number of hours he must work next month to make a profit of $300.
Write an algebraic expression for each situation.

9. the cost of \(x\) apples at $0.49 each
10. the number of hits a 0.306 batter gets in \(b\) times at bat
11. the number of minutes it takes to read \(p\) pages at 10 minutes per page
12. the money left on a $20 gift card after spending \(y\) dollars
13. the distance traveled over \(t\) hours at \(r\) miles per hour

Evaluate each algebraic expression for \(a = 12\) and \(b = 3\).

14. \(a - 2\)  
15. \(5a\)  
16. \(a + b\)  
17. \(\frac{3a}{2b}\)

Evaluate each algebraic expression for \(d = \frac{3}{4}, e = \frac{4}{9},\) and \(f = \frac{1}{2}\).

18. \(d + f\)  
19. \(de\)  
20. \(f - e\)  
21. \(4d + 2f\)

Write a situation that could describe each algebraic expression.

22. \(a + 24\)  
23. \(365 - d\)  
24. \(7w\)  
25. \(\frac{m}{35}\)

26. At a craft store, each package of beads costs $3.95.
   a. Write an algebraic expression for the cost for \(p\) packages of beads.
   b. Amy gives the sales clerk $20 for \(p\) packages of beads. Write an algebraic expression to represent Amy's change.
   c. What is the greatest number of packages that Amy can buy with $20?

For Exercises 27–34, decide which operation is needed to isolate the variable. Solve the equation.

27. \(a + 6 = 14\)  
28. \(b - 3 = 9\)
29. \(4d = 12\)  
30. \(7 + t = 15\)
31. \(\frac{x}{2} = 5\)  
32. \(\frac{n}{5} = 6\)
33. \(y - 13 = 29\)  
34. \(11h = 132\)

35. Greg counted 11 people who got on the bus at the last stop. Now every seat is filled. How many people were on the bus before the stop if the bus has seats for 42 people?

36. There are four dozen daisies in a vase. If every person receives three daisies until the daisies are gone, how many people will get daisies?

37. A flower garden has 18 square feet of space. A packet of seeds fills 2 square feet. How many packets of seeds are needed to fill the garden?

38. Becky wants to solve the equation \(3x = 18\). She says that \(18 - 3 = 15\), so \(x = 15\). Explain to Becky how to find the correct answer.
For Exercises 39 and 40, write an algebraic expression.

39. seven times a number
40. a number of objects is split into 6 equal groups

For Exercises 41–44, evaluate each expression.

41. \(12x\) for \(x = 7\)
42. \(112 ÷ x\) for \(x = 7\)
43. \(2x - 3\) for \(x = 9\)
44. \(\frac{x}{3} + 6\) for \(x = 400\)

For Exercises 45 and 46, use the information below.

Jennifer pays an $80 down payment on a violin. She will pay the rest off at $20 a week.

45. What expression can Jennifer use to represent this situation? Explain what the variable represents.

46. If the violin costs $400, how long will it take Jennifer to pay for it?

For Exercises 47–50, use this information and advertisement.

Grace and Tina are planning a canoeing trip. They are deciding whether they should rent 2 single-seat canoes or 1 two-seat canoe. Also, they will need to rent a canoe carrier.

<table>
<thead>
<tr>
<th>Two-Seat Canoes</th>
<th>Single-Seat Canoes</th>
</tr>
</thead>
<tbody>
<tr>
<td>$45 per day</td>
<td>$25 per day</td>
</tr>
<tr>
<td>$25 canoe carrier per trip</td>
<td>Free canoe carrier</td>
</tr>
</tbody>
</table>

47. What information is needed to find the cost of renting the canoes?

48. What expression can Grace and Tina use to find the cost of renting a two-seat canoe?

49. What expression can they use to find the cost of renting 2 single-seat canoes?

50. For a 4-day trip, which type of canoe should Grace and Tina rent if they want to spend the least amount of money? Explain.
For Exercises 51 and 52, use the information below.

Roses cost $20 per dozen. The delivery fee for any order is $8.

51. If \( r \) represents the number of dozen roses, write an expression to represent the cost of \( r \) dozen roses including delivery.

52. What is the total cost of having 3 dozen roses delivered?

53. Lilah buys 2 board games. Each board game costs \( g \) dollars. She has a $6 credit from a previous purchase. Write an expression to represent the amount Lilah pays for the two games.

For Exercises 54–55, use the information and table below.

The Johnson family is having a party. They need to buy paper plates, plastic forks, and plastic spoons. The table shows how each is sold.

<table>
<thead>
<tr>
<th>Item</th>
<th>Number in a Package</th>
</tr>
</thead>
<tbody>
<tr>
<td>Paper Plates</td>
<td>( x )</td>
</tr>
<tr>
<td>Plastic Forks</td>
<td>( y )</td>
</tr>
<tr>
<td>Plastic Spoons</td>
<td>( z )</td>
</tr>
</tbody>
</table>

54. a. The Johnsons bought 3 packages of paper plates and 4 packages of plastic forks. Write an expression to represent the number of paper plates and plastic forks they bought altogether.
   b. There are 100 paper plates in a package and 50 plastic forks in a package. Find how many plates and forks the Johnsons bought.

55. a. Write an expression to find the number of packages needed to buy 375 plastic spoons.
   b. If there are 75 plastic spoons in a package, how many packages were bought?

For Exercises 56 and 57, use the information below.

Luz is making a tabletop design with tiles. For each step in his pattern he can determine the number of tiles he needs by multiplying the step number by 6 and subtracting 2.

56. How many tiles will he need for the sixteenth step?

57. In which step will Luz need to use exactly 70 tiles?
For Exercises 58–61, use the figures shown below.

58. Copy and complete the table to show how many tiles are in each figure.

<table>
<thead>
<tr>
<th>Stage</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
</tr>
</thead>
<tbody>
<tr>
<td>Number of Tiles</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

59. Let $s$ be the stage number. Write a rule for finding the number of tiles needed for any stage in the pattern.

60. How many tiles are needed for the ninth stage?

61. If you have 100 tiles, what is the largest stage you can complete?

For Exercises 62–64, use the Input-Output table below.

<table>
<thead>
<tr>
<th>Input ($x$)</th>
<th>16</th>
<th>24</th>
<th>40</th>
<th>52</th>
</tr>
</thead>
<tbody>
<tr>
<td>Output ($y$)</td>
<td>2</td>
<td>4</td>
<td>8</td>
<td>11</td>
</tr>
</tbody>
</table>

62. Write a rule that can be used to find $y$ if $x$ is given.

63. If you know $x = 100$, how can you find $y$? Give the value.

64. If you know $y = 20$, how can you find $x$? Give the value.

65. **Multiple Choice** Matt earns $10 for mowing his neighbor’s lawn and $5 an hour for cleaning out the garage. The equation $e = 10 + 5h$ can be used to find his earnings. If he earned $40, how many hours did it take him to clean out the garage?

   - A. 4
   - B. 6
   - C. 10
   - D. 210

66. A field is twice as long as it is wide. Let $w$ represent the field’s width.
   a. Write an expression to represent the length of the field.
   b. The field is 40 meters wide. What is the field’s length?
For Exercises 67 and 68, find the value of $x$.

67. $7.2 + x = 14.1$

68. $\frac{3}{10} = x + \frac{1}{3}$

69. Five friends ate lunch at a restaurant. They had a coupon for $20 off their total bill. The group’s total came to $20 after they used the coupon.
   a. Each person’s lunch had the same price. Write an equation that can be used to determine the price of one lunch.
   b. Solve the equation.

70. The table shows the relationship between the number of melons bought and the total cost.

<table>
<thead>
<tr>
<th>Number Bought</th>
<th>Total Cost</th>
</tr>
</thead>
<tbody>
<tr>
<td>10</td>
<td>$25</td>
</tr>
<tr>
<td>15</td>
<td>$37.50</td>
</tr>
<tr>
<td>30</td>
<td>$75</td>
</tr>
</tbody>
</table>

   a. Write an expression to find the total cost of buying any number of melons.
   b. Sheila is going to buy 62 melons for a banquet. What will be the total cost?

71. Kelly is $x$ years old. Mike is 2 years older than twice Kelly’s age. The sum of Kelly’s and Mike’s ages is 26. How old are Kelly and Mike?

For Exercises 72–78, name the property illustrated in each equation.

72. $0.85 + (3.5 + 4.15) = (0.85 + 3.5) + 4.15$

73. $3d - 15 = 3(d - 5)$

74. $0 + (-1.6) + 2.4 = -1.6 + 2.4$

75. $\frac{1}{2} \times \frac{2}{1} \times \frac{1}{4} = 1 \times \frac{1}{4}$

76. $15(2c - 8) = 30c - 120$

77. $-3.2 + (-8.5x) = -8.5x + (-3.2)$

78. $123 + (-43) + 0 + (-15) = 123 + (-43) + (-15)$
79. A carpenter cuts lengths of wood into equal 3-ft. sections. Write an expression to represent the total length of wood the carpenter needs to make \( n \) sections. Evaluate the expression for \( n = 7, 10, \) and 15.

For Exercises 80–82, simplify each expression. Use a property or operation to justify each step.

80. \(-4 + \frac{5}{2} + \frac{6}{5} + \frac{2}{5} + \frac{4}{5}\)
81. \(5m + 6 + 3(m + 2)\)
82. \(-2\left(\frac{1}{2}k + \frac{1}{3}\right) + 6 + \frac{2}{3}\)

83. Copy and complete the table for the given \( x \)-values.

<table>
<thead>
<tr>
<th>( x )-value</th>
<th>( 3(x + 4) )</th>
<th>( 3x + 4 )</th>
<th>( 3x + 12 )</th>
<th>( 2(x + 6) + x )</th>
<th>( 5x + (2x + 12) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>5</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>10</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Which of the expressions in the top row are equivalent?

84. Multiple Choice  Which expression is not equivalent to the others?
   A. \(6(x - 2)\)
   B. \(2(x - 6) + 4x\)
   C. \(6x - 12\)
   D. \(7x - (2x + 12)\)

For Exercises 85–88, find the equivalent expression from the box at the right.

85. \(c + c + c\)  
86. \(4c - 2 - 3c + 16\)  
87. \(c + c + c + c + 2 + 4\)  
88. \(3c + 6c - 8c\)  

a. \(c\)  
b. \(3c\)  
c. \(c + 14\)  
d. \(4c + 6\)
89. Pat earns $9 per hour working as a lifeguard.
   a. Write an algebraic equation to represent the relationship between the number of hours Pat works, and the amount that she earns.
   b. Identify the dependent and independent variables in the equation.
   c. Use your equation to complete the table to show the money earned for working during the week.

<table>
<thead>
<tr>
<th>Day</th>
<th>Monday</th>
<th>Tuesday</th>
<th>Wednesday</th>
<th>Thursday</th>
<th>Friday</th>
<th>Saturday</th>
</tr>
</thead>
<tbody>
<tr>
<td>Hours Worked</td>
<td>8</td>
<td>6</td>
<td>7</td>
<td>8</td>
<td>4</td>
<td>5</td>
</tr>
<tr>
<td>Earnings (in $)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

For Exercises 90–93, write an algebraic equation to relate the quantities.

90. Michael rides his bike at an average speed of 18 mi/hr for \( t \) hours. He travels a total distance of \( d \) miles.

91. Susan is \( y \) years old. She is 8 years younger than her brother, who is \( x \) years old.

92. The circumference, \( c \), of any circle is \( 2\pi \) times its radius, \( r \).

93. An object’s mass in kilograms, \( w \), is its mass in grams, \( z \), divided by 1,000.
94. **Multiple Choice** The solutions to which equation are shown on this graph?

![Graph showing a linear relationship between p and g.]

A. \( g = \frac{p}{2} \)

B. \( p = \frac{g}{2} \)

C. \( p = g - 2 \)

D. \( g = p + 2 \)

95. The table shows the prices of some produce at a farmers’ market.

<table>
<thead>
<tr>
<th>Produce</th>
<th>Price</th>
</tr>
</thead>
<tbody>
<tr>
<td>Apples</td>
<td>$7 per basket</td>
</tr>
<tr>
<td>Pears</td>
<td>$7 per basket</td>
</tr>
<tr>
<td>Corn</td>
<td>$0.75 per ear</td>
</tr>
<tr>
<td>Asparagus</td>
<td>$3.50 per bundle</td>
</tr>
<tr>
<td>Broccoli</td>
<td>$2.50 per bag</td>
</tr>
</tbody>
</table>

Write two equivalent algebraic expressions to represent the total cost.

a. some ears of corn

b. 3 baskets of apples and some baskets of pears

c. some baskets of apples, 2 baskets of pears, and 4 ears of corn
At a Glance

PACING 4 days

CC Investigation 3: Integers and the Coordinate Plane

Mathematical Goals

- Recognize that numbers with opposite signs are located on opposite sides of 0 on the number line, and that the opposite of the opposite of a number is the number.
- Understand how signs of the numbers in an ordered pair indicate the point’s location in a quadrant of the coordinate plane.
- Recognize how points indicated by ordered pairs that differ only by signs relate to reflections across one or both axes.
- Find and graph rational numbers on a number line and ordered pairs of rational numbers on a coordinate plane.
- Understand how a rational number’s absolute value is its distance from 0 on the number line.
- Interpret absolute value as magnitude for a quantity in a real-world situation and distinguish comparisons of absolute value from statements about order.
- Solve problems, including distance problems involving points with the same x-coordinate or same y-coordinate, by graphing points in all quadrants of the coordinate plane.
- Write an inequality to represent a real-world situation.
- Graph the solution to an inequality and recognize that inequalities of the form \( x > c \) or \( x < c \) have an infinite number of solutions.
- Draw polygons in the coordinate plane when given coordinates for the vertices.

Teaching Notes

In this investigation, students will extend their previous understandings of number and the ordering of numbers to the full system of rational numbers, which includes negative rational numbers, and in particular negative integers. They will reason about the order and absolute value of rational numbers and about the locations of points in all four quadrants of the coordinate plane. Students will apply their knowledge of the coordinate plane to plot vertices of polygons and find the lengths of their horizontal or vertical sides.

Vocabulary
- opposite
- integer
- rational number
- absolute value
- coordinate plane
- quadrants
- ordered pairs
- origin
- inequality

Materials
- grid paper
Problem 3.1

Before Problem 3.1, review the number line in the investigation and ask:
• How far is the number $-2$ from 0 on the number line? (2 units)
• How far is the number $+2$ from 0 on the number line? (2 units)
• Name the other opposites you see on the number line. ($-5$ and $+5$, $-4$ and $+4$, $-3$ and $+3$, $-1$ and $+1$)
• How could you use an integer to represent a temperature of twelve degrees below zero? a loss of thirty dollars? ($-12$; $-30$)

During Problem 3.1, ask:
• What integer will you use to represent each player's starting position? (0)
• How will you represent steps forward? (Move to the right on the number line.)
• How will you represent steps backward? (Move to the left on the number line.)

Problem 3.2

Before Problem 3.2, ask: How do you decide where to place numbers on a number line? (A number is always greater than numbers to its left on the number line, and less than all numbers to its right.)

During Problem 3.2 A, ask: Which two numbers are the same distance from 0 on the number line? ($-2$ and $2$)

During Problem 3.2 B, ask:
• How do you know where to show Sahil's number, $\frac{-3}{5}$, on the number line? (Find an equivalent fraction for $\frac{-3}{5}$ with a denominator of 10: $\frac{-3}{5} = \frac{-6}{10}$.)
• How do you know where to show Emily's number, $\frac{2}{3}$, on the number line? (The decimal equivalent to $\frac{2}{3}$ is $0.6\overline{6}$, so show it between $\frac{6}{10}$ and $\frac{7}{10}$.)

During Problem 3.2 C, ask:
• What is the distance between tick mark on the number line? (0.5 or $\frac{1}{2}$)
• How do you know where to show the numbers 2.4 and 0.8 on the number line? (2.4 is between 2 and 2.5, so show it in that area closer to 2.5; 0.8 is between 0.5 and 1, so show it there.)
Problem 3.3

Before Problem 3.3, ask: Is the point (5, 3) the same as the point (3, 5)?
(No; Locate (5, 3) by going right from the origin 5 units and then up 3 units. Locate (3, 5) by going right from the origin 3 units and then up 5 units.)

During Problem 3.3 A, ask:
- What characteristic is shared by all points on the x-axis? (Every point on the x-axis has a y-coordinate of zero.) by all points on the y-axis?
  (Every point on the y-axis has an x-coordinate of zero.)
- What characteristic is shared by all points in quadrants I and III? (Their coordinates have the same sign.) by all points in quadrants II and IV?
  (Their coordinates have opposite signs.)

Problem 3.4

Before Problem 3.4, ask:
- Think about a vertical line that crosses the x-axis at 2. What do you know about the x-coordinate of every point on that line? (It is 2.)
- Think about a horizontal line that crosses the y-axis at ~4. What do you know about the y-coordinate of every point on that line? (It is ~4.)

During Problem 3.4 A, ask: How can you use the grid on the coordinate plane to find the length of a line segment that is drawn on it? (Count the number of units on the grid.)

During Problem 3.4 B, ask:
- How far is point C from the y-axis? (9 units)
- How far is point D from the y-axis? (4 units)
- How can you use these distances to tell how long the line segment connecting points C and D is? (Add the distances.)

After Problem 3.4, ask: What general rule can you use to find the distance between two points on a horizontal line? (Find the number of units between the x-coordinates.) between two points on a vertical line? (Find the number of units between the y-coordinates.)

Problem 3.5

Before Problem 3.5, ask:
- What do the x-coordinates of the vertices at (3, 4) and (3, 1) tell you about the line segment that connects them? (It is vertical.)
- What do the y-coordinates of the vertices at (3, 1) and (~1, 1) tell you about the line segment that connects them? (It is horizontal.)

During Problem 3.5 A, ask: When you write a subtraction sentence, does it matter which coordinate you subtract from the other? Why or why not? (It does not matter, since the distance is the absolute value of the difference.)

During Problem 3.5 B, ask:
- What is the side length of a square with a perimeter of 16 units? (4 units)
- What are the coordinates of 4 points that are each 4 units from (2, ~2)? ((~2, ~2), (6, ~2), (2, ~6), (2, 2))
**Problem 3.6**

*Before* Problem 3.6, review the inequality symbols, and ask:

- Does an inequality symbol open to the lesser or to the greater number? (greater number)
- How is the solution to the inequality $c \geq 5$ different than the solution to the inequality $c > 5$? (For $c \geq 5$, 5 is a solution, and in $c > 5$, it is not.)

*During* Problem 3.6, ask: What does the arrow on the graph of an inequality represent? (The solutions include all numbers in that direction, without end.)

**Summarize**

To summarize the lesson, ask:

- What do you call numbers with different signs that are the same distance from 0 on the number line? (opposites)
- What is the opposite of the opposite of 3? (3)
- In what quadrant of the coordinate plane is the point $3, -4$? (quadrant IV)
- What is the distance of the point $-5$ from 0 on the number line? (5 units)
- How can you find the distance between two points on a horizontal line? (Find the number of units between their $x$-coordinates.)
- How many solutions are there to the inequality $p > 8$? (an infinite number)

Students in the CMP2 program will further study standards 6.NS.6.a, 6.NS.6.b, 6.NS.6.c, 6.NS.7.c, 6.NS.7.d, and 7.NS.8 in the Grade 7 Unit Accentuate the Negative.

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**Assignment Guide for Investigation 3**

Problem 3.1, Exercises 1–9
Problem 3.2, Exercises 10–13
Problem 3.3, Exercises 14–37
Problem 3.4, Exercises 38–47
Problem 3.5, Exercises 48–55
Problem 3.6, Exercises 56–69

**Answers to Investigation 3**

**Problem 3.1**

A. 1. 0

2.

```
5 4 3 2 1 0 +1 +2 +3 +4 +5
```

3. Sahil

4. Emily and Juan

B. 1.

```
5 4 3 2 1 0 +1 +2 +3 +4 +5
```

2. No, Emily’s and Juan’s positions now are $-1$ and $+5$, which are not opposites.

3. The numbers are increasing.

4. There is no other player at the opposite of Cora, since Cora is at 0.

C. Sahil’s new position is $+1$, so he took 4 steps forward.
Problem 3.2
A. 1.

Sahil, N; Cora, I; Emily, C; Juan, E

2. Sahil, N; Cora, I; Emily, C; Juan, E

3. No students have numbers with the same absolute value.

B. 1.

Juan, M; Sahil, A; Emily, T; Cora, H

2. Juan, M; Sahil, A; Emily, T; Cora, H

C. 1.

Sahil, Cora, Emily, Juan

Problem 3.3
A. 1. an umbrella

2. Points (2, 3) and (4, 1) are in quadrant I; points (–2, 3) and (–4, 1) are in quadrant II; points (–5, –1), (–3, –1), (–1, –1), (–3, –4), (–2, –5), and (–1, –4) are in quadrant III; points (1, 1), (3, 1), and (5, 1) are in quadrant IV; (0, 0) is the origin, points (–4, 0), (–2, 0), (2, 0), and (4, 0) are on the x-axis, and point (0, –4) is on the y-axis.

3. Check students’ work.

B. quadrants I or III, or at the origin

C. 1. The signs are different.

2. They both are 4 units from the y-axis, on opposite sides of it.

3. reflection across the y-axis, translation 8 units left, or 180° rotation around (0, 2)

4. They both are 2 units from the x-axis, on opposite sides of it.

Problem 3.4
A. 1. Their y-coordinates are the same, and their x-coordinates are different.

2. horizontal

3. 6 units

4. a. 2 units

b. 4 units

c. Add the points’ distances from the y-axis to find the length of the segment joining them.

B. 1. Their y-coordinates are the same.

2. 13 units

C. 1. vertical

2. Find the sum of the points’ distances from the x-axis; 9 + 6 = 15 units.

D. 1. 5 + 3 = 8 units

2. No, the line segment connecting points S and N is not vertical or horizontal.

Problem 3.5
A. 1.

2. rectangle

3. They have the same x-coordinates.

4. 4 – 1 = 3

5. 3 – (–1) = 4; 4 – 1 = 3, 3 – (–1) = 4

6. 14 units
B. 1. Sample answer: (2, 2), (−2, 2), (−2, −2)

2.

3. No, the point (2, −2) could be any of the 4 vertices of a square with side length 4 units.

Problem 3.6

A. 1. \( t > −3; t \) represents the temperature in degrees Celsius.

2. An open circle indicates that −3 is not included in the solution.

3. The arrow pointing to the right indicates that there are an infinite number of solutions to the inequality.

B. 1. \( t \leq 30; t \) represents the time it will take in minutes.

2. A closed circle indicates that 30 is included as part of the solution.

3. Yes, values of \( t \) less than 0, such as −5, do not make sense because \( t \) represents time, and negative time does not make sense.

Exercises

1. −1 and 1; −4 and 4

2. −3 and 3

3. −10 and 10

4. −10 and 10, −5 and 5

5. a. 6

6. a. 20

7. a. 12

8. a. 0

9. a. 0; Sample answer: \( a = −5, e = 5 \)

10. B

11. Sample answer: \( −1\frac{1}{2} \)

12. Sample answer: 17.5

13. a. \( \frac{1}{5}, \frac{1}{4}, \frac{1}{3}, \frac{1}{2} \)

b. smaller

c. No, as the denominator of a negative fraction increases, the value of the fraction also increases.

14–21.

22. quadrant III

23. quadrant II

24. y-axis

25. quadrant I

26. quadrant III
27. quadrant IV
28. quadrant II
29. x-axis
30. quadrant I
31. quadrant III
32. D
33. Sample answer: (3, −3); acceptable answers should include a positive x-coordinate and a negative y-coordinate.
34. (0, 0)
35. No, (−5, 5) is in quadrant II, and (5, −5) is in quadrant IV.
36. If the x-coordinate is 0, then the point lies on the y-axis. If the y-coordinate is 0, then the point lies on the x-axis.
37. No, the origin, and points on either axis do not lie in any quadrant.
38. horizontal; 7 units
39. vertical; 12 units
40. horizontal; 27 units
41. neither
42. vertical; 7 units
43. vertical; 7 units
44. horizontal; 18.5 units
45. vertical; 4.8 units
46. horizontal; 7 units
47. a. 6 units
b. 8 units
48–51. part (a) answers:

48. b. (3, 2)
c. (3, 3) and (6, 3)
d. (3, 1) and (6, 1)
49. b. (0, 1)
c. (1, 4) and (1, 1)
d. (0, 4) and (0, 1)
50. b. (−1, 2)
c. (−5, 4) and (−1, 4)
d. (−5, 2) and (−1, 2)
51. b. (0, 4)
c. (0, 1) and (4, 1)
d. (0, 0) and (4, 0)
52. (−4, 5) and (4, −5)
53. (3, −3) and (−3, 3)
54. (1, 2) and (−1, −2)
55. (5, 5) and (−5, −5)
56. n > 7
57. n ≤ −6
58. g ≤ 50
59. s ≥ 90
60. s > 150
61. t ≥ −20
62. D

63.

64.

65.

66.
67. $t < 45$; an open circle shows that 45 is not part of the solution, so that Carlos can be at the movie before it starts.

68. $t < -10$; an open circle shows that -10 is not part of the solution, so only temperatures below -10°C are excluded.

69. $x \leq 6.75$; Possible answers: $4.50, 5.50, 6.50$
Additional Practice

1. Nadia recorded the daily low temperatures, in degrees Celsius, on this vertical number line.
   a. On which days were the temperatures opposites?
   b. The low temperature increased by 2°C between which two days?
   c. Find the absolute value of the temperature each day and order the days from least absolute value to greatest.

2. Andrew plotted the three points on the coordinate plane.
   a. Give the coordinates of points A, B, and C.
   b. How much farther is it from point B to point A than from point B to point C? Explain.
   c. Andrew plots point D and connects the four points to form a rectangle. What is the perimeter of rectangle ABCD? Explain.

3. Visitors at a theme park must be at least 48 inches tall to ride a new ride.
   a. Write an inequality that represents the height requirement. Explain what the variable represents.
   b. Graph the inequality on a number line. Explain your choice of a closed circle or open circle.
   c. How many values are represented by the inequality? Explain how this is shown on the graph.
   d. Are there any numbers included in the solution that don’t make sense for the situation? If so, give an example and explain why that solution does not make sense.
Skill: Identify Ordered Pairs

Integers and the Coordinate Plane

Give the ordered pair for each point.

1. point A  
2. point B  
3. point C  
4. point D  
5. point E  
6. point F  
7. point G  
8. point H  
9. point I  
10. point J  
11. point K  
12. point L

Skill: Interpret Graphs of Inequalities

Write an inequality represented by each graph.

13.  
14.  
15.  
16.  
17.  
18.  

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Check-Up

Integers and the Coordinate Plane

1. There are four colored pieces on a game board. In the first round, the red piece moves 3 spaces forward, the yellow piece moves 2 spaces back, the blue piece moves 2 spaces forward, and the green piece does not move.
   a. What integer describes the yellow piece’s position in the game?
   b. Draw a number line. Represent each piece’s position on the number line.
   c. Which pieces are represented by opposites?
   d. In the second round, all of the pieces move back 2 spaces. Which piece now is farthest from its starting point? Which piece is closest?

2. Carmen invests in stocks. The table shows the changes in her stocks’ prices over the last month.

<table>
<thead>
<tr>
<th>Stock</th>
<th>Price Change (in $)</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>+2.45</td>
</tr>
<tr>
<td>B</td>
<td>+7.16</td>
</tr>
<tr>
<td>C</td>
<td>-8.32</td>
</tr>
<tr>
<td>D</td>
<td>-0.98</td>
</tr>
</tbody>
</table>

a. Carmen wants to order her stocks from biggest loss to biggest gain using a number line. Estimate each stock’s position on this number line to show how Carmen should order them.

b. Next, Carmen wants to order the stocks from least price change to greatest price change. Explain how Carmen could use absolute value to order the stocks.
Check-Up (continued)

Integers and the Coordinate Plane

3. Jin plotted the four points on the coordinate plane.
   a. How are the coordinates of points $A$, $B$, and $D$ alike, and how are they different?

   b. What transformations can be used to transform the location of point $A$ to point $B$?

   c. What is the perimeter of the rectangle that could be drawn with points $A$ and $C$ as opposite, diagonal vertices? Explain how you found your answer.

4. Harrison’s school is selling boxes of greeting cards to raise money for new computers for the school. Students can win rewards by raising certain amounts of money.

<table>
<thead>
<tr>
<th>If you raise . . .</th>
<th>then you win . . .</th>
</tr>
</thead>
<tbody>
<tr>
<td>$30 or more</td>
<td>a t-shirt.</td>
</tr>
<tr>
<td>$60 or more</td>
<td>a t-shirt and a tote bag.</td>
</tr>
<tr>
<td>$90 or more</td>
<td>a t-shirt, a tote bag, and a raffle ticket for a computer.</td>
</tr>
</tbody>
</table>

   a. Harrison wants at least a t-shirt and a tote bag. Write an inequality to show how much money Harrison needs to raise.

   b. Graph the inequality. Explain what your graph shows.

   c. Rebecca raised enough to get a t-shirt, but not enough to get a raffle ticket. Make a graph on a number line to show the possible amounts Rebecca raised.
Investigation 3: Integers and the Coordinate Plane

Negative numbers are needed when quantities are less than zero, such as very cold temperatures. Temperatures in winter go below 0°F in some locations. An altitude of 0 feet is referred to as sea level, but there are places in the world that are below sea level.

The counting numbers and zero are called whole numbers. The first six whole numbers are 0, 1, 2, 3, 4, and 5. You can extend a number line to the left past zero.

The opposite of a positive number is a negative number. For example, the number \(-2\) is the opposite of \(+2\). The set of whole numbers and their opposites are called integers.

Emily, Juan, Sahil, Cora, and Austin play a Question and Answer game. A player steps forward for a correct answer, but steps backward for an incorrect answer. During the first round, Sahil takes five steps backward. Juan takes three steps forward. Emily takes three steps backward. Austin does not move. Cora takes two steps backward.

A. 1. Which integer describes Austin’s position in the game?
   2. Draw a number line. Represent each player’s position on the number line.
   3. Who is in last place?
   4. Which players are represented by opposites?

B. 1. In the next round each player moves two steps forward. Place all five players on a new number line.
   2. Are any players who were opposites before still opposites now? Why or why not?
   3. What does it mean when you read the numbers on the number line from the left to the right?
   4. Which player is at the opposite of Cora? Explain.

C. In the final round, Emily stays in the same place, and Sahil is at her opposite. How many steps did Sahil take in the final round?
Rational numbers are numbers that can be expressed as one integer divided by another non-zero integer. Examples of rational numbers are $\frac{3}{4}$, $-\frac{7}{8}$, $\frac{3}{7}$, and 0.75.

The absolute value of a number $a$, represented as $|a|$, is the distance between the number $a$ and zero on the number line. Because distance is a measurement, the absolute value of a number is never negative.

Opposites, like $-3$ and $3$, have the same absolute value because they are the same number of units from zero.

Emily, Cora, Sahil, and Juan are playing another game. Each player gets a card with a number on it. The reverse side of the card contains a hidden letter. The four players line up on a number line. If they are correct, the hidden letters spell a word.

A. 1. For Round 1, Emily has 0, Sahil has $-3$, Juan has 2, and Cora has $-1$. Use a number line to show how the students should line up.

2. When they reveal their letters, they spell the word NICE. Assign each letter to the proper student.

3. Which students have numbers that have the same absolute value?

B. 1. For Round 2, Emily has $\frac{2}{3}$, Sahil has $\frac{7}{5}$, Juan has $-\frac{1}{2}$, and Cora has 1. Use a number line to show how the students should line up.

2. When they reveal their letters, they spell AMTH. The students then line up from least to greatest using the absolute values of their numbers to spell a word. Assign each letter to the proper student.

C. For Round 3, Emily has 2.4, Sahil has 5, Juan has 0.8, and Cora has $-2\frac{1}{2}$. 

1. Use a number line to show how the students should line up.

2. Next, the students line up from greatest to least using the absolute values of their numbers. Give the order of the students in line.
A coordinate plane, or Cartesian plane, is formed by two number lines that intersect at right angles. The horizontal number line is the $x$-axis, and the vertical number line is the $y$-axis. The two axes divide the plane into four quadrants.

All points in the plane can be named using coordinates, or ordered pairs written in the form $(x, y)$. The first number is the $x$-coordinate. The second number is the $y$-coordinate. The point of intersection of the two axes is called the origin $(0, 0)$. The origin is labeled with the letter $O$.

**Problem 3.3**

**A.** Cora has a puzzle for Austin. “What goes up a chimney down, but can’t go down a chimney up?”

1. Help Austin find the answer by plotting and connecting these points in order on a coordinate plane. What is the answer?

   Start $(-5, -1), (-4, 1), (-2, 3), (2, 3), (4, 1), (5, -1), (4, 0), (3, -1), (2, 0), (1, -1), (0, 0), (-1, -1), (-2, 0), (-3, -1), (-4, 0), (-5, -1)$. End.

   Then start at $(0, 0), (0, -4), (-1, -5), (-2, -5), (-3, -4)$. End.

2. Name the quadrant, axis, or origin where each point is located in your drawing.

3. Make a picture puzzle on the coordinate plane for a classmate to solve. Use at least one point in each quadrant.

**B.** A point has coordinates with the same sign. Where could the point be located?

**C.** Look at points $(4, 2), (-4, 2)$, and $(4, -2)$.

1. How are the coordinates of the points different?

2. Compare the positions of points $(4, 2)$ and $(-4, 2)$ relative to the $y$-axis.

3. What transformations could be used to transform a point at $(4, 2)$ to a point at $(-4, 2)$?

4. Compare the positions of points $(4, 2)$ and $(4, -2)$ relative to the $x$-axis.

5. What transformations could be used to transform a point at $(4, 2)$ to a point at $(4, -2)$?
A. Consider the points $A(-2, 3)$ and $B(4, 3)$.

1. What is the same about the coordinates of points $A$ and $B$? What is different?

2. Plot the points on a coordinate grid.
   Draw the line segment joining points $A$ and $B$. Is the line segment *horizontal* or *vertical*?

3. Using the coordinate grid, what is the length of the line segment joining points $A$ and $B$?

4. Look at the $x$-coordinates of points $A$ and $B$.
   - a. How far is point $A$ from the $y$-axis?
   - b. How far is point $B$ from the $y$-axis?
   - c. Explain how you can use the points’ distances from the axis to find the length of the line segment joining them.

B. 1. How can you tell when two points will form a horizontal line segment?

2. What is the length of the horizontal line segment joining points $C(-9, 205)$ and $D(4, 205)$?

C. 1. Is the line segment connecting points $T(-8, -9)$ and $M(-8, 6)$ *horizontal*, *vertical*, or *neither*?

2. How can you use the coordinates of points $T$ and $M$ to find the length of the line segment joining points $T$ and $M$?

D. 1. Find the length of the line segment joining points $F(a, 5)$ and $G(a, -3)$.

2. Can you use the same method to find the length of the line segment joining $S(1, 1)$ and $N(3, 4)$? Explain.
You can connect the points plotted on a coordinate plane to draw polygons. Use the coordinates of the polygon’s vertices to find the lengths of the polygon’s sides.

**Problem 3.5**

Emily and Juan play a game where each gives the coordinates of some points, and the other guesses the polygon that is made when the points are plotted and connected on a coordinate plane.

**A.** Emily gives Juan the coordinates for the vertices of a quadrilateral: (3, 4), (3, 1), (−1, 1), (−1, 4).

1. Graph the points on a coordinate plane.
2. Connect the points in order. What quadrilateral did you draw?
3. What do you notice about the coordinates of the vertices located at (3, 4) and (3, 1)?
4. Write a subtraction sentence that you can use to find the length of the side of the polygon between the vertices located at (3, 4) and (3, 1).
5. Write subtraction sentences that you can use to find the lengths of the other sides of the polygon.
6. What is the perimeter of the polygon?

**B.** Juan begins to give Emily the vertices of a square with a perimeter of 16 units by giving her the coordinate (2, −2).

1. Give a set of possible coordinates for the other 3 vertices of the square.
2. Graph the square on a coordinate plane.
3. Is that the only square that Juan could have been describing? Explain your answer.
An inequality is a mathematical sentence that compares two quantities that are not equal. Use the following symbols to represent inequalities.

- $<$ means “is less than”
- $\leq$ means “is less than or equal to”
- $>$ means “is greater than”
- $\geq$ means “is greater than or equal to”

An inequality can be graphed on a number line. Use an open circle when graphing an inequality with $<$ or $>$, because the number is not part of the solution. Use a closed circle for $\leq$ and $\geq$, because the solution includes the number.

\[ p > 2 \quad \quad g \leq 4 \]

Juan and Emily are playing a game with inequalities called More or Less. Each draws a card that describes a situation, and must write and graph an inequality to represent the situation.

**A.** Juan draws a card that reads, “The temperature was higher than $–3\,{}^\circ C$.”

1. What inequality should Juan write? Explain what the variable represents.
2. Graph the inequality on a number line. Explain your choice of a closed circle or an open circle.
3. How many solutions are represented by the inequality? Explain how this is shown on the graph.

**B.** Emily draws a card that reads, “It will take 30 minutes or less.”

1. What inequality should Emily write? Explain what the variable represents.
2. Graph the inequality on a number line. Explain your choice of a closed circle or an open circle.
3. Are there numbers included in the solution that do not make sense for the situation? If so, give an example and explain why that solution does not make sense.
**Exercises**

For Exercises 1–4, graph each integer on a number line. Then identify any opposites.

1. \(-1, 4, 2, -4, 3, 1\)
2. \(2, 0, -3, 4, -1, 3\)
3. \(-5, 10, -2, 4, 0, -10\)
4. \(-5, 8, -7, -10, 5, 10\)

5. Use an integer to represent each play in a football game.
   a. The fullback carries the ball for a gain of 6 yards.
   b. The quarterback is sacked for a loss of 3 yards.
   c. The play stops at the line of scrimmage for no gain.

6. Use an integer to represent each change to a bank account.
   a. A deposit of $20 is made on Monday.
   b. A check for $4 is written on Tuesday.
   c. A check for $6 is written on Wednesday.
   d. No transactions are made on Thursday.

7. Use an integer to represent each position of an elevator.
   a. The elevator leaves the ground floor and arrives at the 12th floor.
   b. The elevator leaves the ground floor and arrives at the second basement level.
   c. The elevator leaves the ground floor, arrives at the 7th floor, and then travels down 3 floors.

8. Use an integer to represent time in seconds for a space-ship launch.
   a. Lift off.
   b. The countdown begins with 10 seconds before lift off.
   c. The space ship has been in the air for one minute.
   d. Why do you think a launch countdown starts at T-minus ten seconds?

9. Use the number line below.

```
  a   b   c   d   e
```

   a. If \(a\) and \(e\) are opposites, what integer would you use to represent \(c\)?
      Assign integer values to \(a\) and \(e\).
   b. If \(a\) and \(d\) are opposites, is \(c\) positive or negative? Explain.
10. **Multiple Choice** Which list shows the numbers ordered from least absolute value to greatest absolute value?

A. \(-4, -2, 6\)  
B. \(-2, -4, 6\)  
C. \(6, -2, -4\)  
D. \(6, -4, -2\)

11. Write a mixed number that is greater than \(-2\) and less than \(-1\).
12. Write a decimal that is less than \(|-18|\) and greater than 17.

13. a. Order the numbers \(\frac{1}{2}, \frac{1}{3}, \frac{1}{4}\), and \(\frac{1}{5}\) from least to greatest.  
b. As the denominator of a fraction increases, does the resulting positive fraction get larger or smaller?  
c. Does your rule apply for \(\frac{-1}{2}, \frac{-1}{3}, \frac{-1}{4}\)? Explain.

For Exercises 14–21, plot the points on a coordinate plane.

14. \(A(-5, 3)\)  
15. \(B(-3, -1)\)  
16. \(C(0, 0)\)  
17. \(D(-1, 0)\)

18. \(E(3, 3)\)  
19. \(F(0, 2)\)  
20. \(G(4, -2)\)  
21. \(H(-2, -4)\)

For Exercises 22–31, identify the location (quadrant number or axis) of each point.

22. \((-36, -11)\)  
23. \((-15, 35)\)  
24. \((0, -100)\)  
25. \((820, 657)\)

26. \((-721, -42)\)  
27. \((549, -90)\)  
28. \((-246, 280)\)  
29. \((333, 0)\)

30. a point with a positive x-coordinate and a positive y-coordinate  
31. a point with a negative x-coordinate and a negative y-coordinate

32. **Multiple Choice** Where does point \(Z(0, 0)\) lie?  
   A. quadrant I  
   B. quadrant II  
   C. quadrant IV  
   D. origin

For Exercises 33–34, write a possible set of coordinates of each point.

33. a point in quadrant IV  
34. a point on both the x- and y-axis

35. Are \((-5, 5)\) and \((5, -5)\) in the same quadrant? Explain.

36. Explain how you can tell whether a point lies on either the x- or y-axis by looking at its coordinates.

37. Sam says that all points on a coordinate plane lie in a quadrant. Do you agree or disagree? Explain.
For Exercises 38–46 below, determine if the line segment joining the two points is horizontal, vertical, or neither. If the points are horizontal or vertical, find the length of the line segment joining the two points.

38. (2, 5), (9, 5)  
39. (4, 0), (4, −12)  
40. (−7.5, −6.25), (19.5, −6.25)

41. \(\left(\frac{1}{2}, \frac{1}{2}\right)\)  
42. (5, 9), (5, 2)  
43. (0, 0), (0, −7)

44. (9.25, 1.5), (−9.25, 1.5)  
45. (−1.2, −1.2), (−1.2, 3.6)  
46. (0, 0), (−7, 0)

47. Use the coordinate grid below.

a. Find the length of a line segment joining points \(U\) and \(V\).

b. Find the length of a line segment joining points \(W\) and \(V\).

For Exercises 48–51, do parts (a)–(d).

a. Graph the given ordered pairs and connect them with a line segment.

b. Find a point that can connect to make a right triangle.

c. Find two points that can connect to make a square.

d. Find two points that can connect to make a rectangle that is not also a square.

48. (3, 0), (6, 0)  
49. (−2, 1), (−2, 4)  
50. (−1, 0), (−5, 0)  
51. (0, −3), (4, −3)

For Exercises 52–55, the two given points are connected to form the diagonal of a rectangle. Find the other two vertices of the rectangle.

52. (4, 5), (−4, −5)  
53. (3, 3), (−3, −3)  
54. (−1, 2), (1, −2)  
55. (−5, 5), (5, −5)
For Exercises 56–61, write an inequality to describe the situation.

56. Ivan chooses a number greater than 7.
57. Ella chooses a number less than or equal to -6.
58. Chen can spend at most $50 on groceries.
59. Juliet wants to get a score of at least 90 on her exam.
60. Michael swam more than 150 laps at practice.
61. The sleeping bag will keep a person warm in temperatures down to -20°F.

62. Multiple Choice  How many solutions are there to the inequality $x \geq 4$?
   A. 4
   B. 0
   C. 5
   D. an infinite number

For Exercises 63–66, graph the inequality on a number line.

63. $b < 2$
64. $-1 \leq j$
65. $b \geq -2$
66. $0 > f$

67. Carlos is trying to get to a movie that starts in 45 minutes. Write an inequality that shows how long Carlos can take if he wants to make it before the start of the movie. Graph the solution. Explain your choice of an open circle or a closed circle in the graph.

68. Etta is planning a trip to Canada, but does not want to visit when the low temperature will be below -10°C. Write an inequality to show temperatures that Etta does not want. Graph the solution. Explain your choice of an open circle or a closed circle in the graph.

69. Ishwar has $6.75 he can spend on lunch. Write an inequality to show how much Ishwar can spend. Graph the solution. Give 3 solutions to the inequality that are not whole numbers.

Notes

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CC Investigation 4: Measurement

Mathematical Goals

- Find the volume of a right rectangular prism with fractional edge lengths by packing it with unit cubes, and show that the volume is the same as found by multiplying the prism’s edge lengths.

- Apply the formulas $V = lwh$ and $V = bh$ to solve real-world problems involving the volumes of right rectangular prisms with fractional side lengths.

- Represent three-dimensional figures using nets made of rectangle and triangles.

- Use nets of three-dimensional figures to solve real-world problems involving surface area.

Teaching Notes

Show students examples of rectangular prisms and pyramids. To represent a three-dimensional shape on a flat page, such as in a book, you must draw a two-dimensional representation of that shape. Read the introduction with students and review the dimensions of three-dimensional figures: length, width, and height. Point out that three-dimensional objects consist of such parts as faces, edges, and vertices, and that by turning the object it can have the appearance of different two-dimensional shapes.

To begin the instruction on surface area, show students a box. Rotate the box to several positions. Review the shape on each side of the box. Ask students to explain how to find the area of one side. Review the formula for area of a rectangle with the class. Show the class a sheet of wrapping paper. Ask the class if they believe there is enough paper to cover the box. Let the students explain how they can find this information. Then develop a definition for surface area using the faces of the box to show the total area that must be covered by the wrapping paper.

Use unit cubes in an exercise to give the students a hands-on experience with cubic measurement and volume. Show that the cubes can be rearranged into numerous configurations without the volume changing.

Note that the solid shapes used in this investigation are all right shapes: the bases of the rectangular and triangular prisms are aligned vertically; the apexes of the pyramids are directly above the centers of the bases.

Vocabulary

- net
- prism
- surface area
- volume
Problem 4.1

Before Problem 4.1, present various models of prisms and pyramids to students and review the faces that make up the figures. Ask:

- What two-dimensional shapes make up the faces of a rectangular prism? (6 rectangles)
- What two-dimensional shapes make up the faces of a square pyramid? (1 square, 4 triangles)

During Problem 4.1 A, ask:

- What two-dimensional shapes make up Ashley’s net? (6 rectangles)
- What are the dimensions of each rectangle in the net? (14 in. × 4 in.; 14 in. × 8 in.; 14 in. × 4 in.; 8 in. × 4 in.; 14 in. × 4 in.)
- What do you notice about how some of the rectangles’ sizes are related? (There are 3 pairs of equal rectangles)

During Problem 4.1 B, ask:

- What two-dimensional shapes make up the faces of the figure? (2 triangles and 3 rectangles)
- What do you notice about the two triangular faces? (They are the same size and shape.)
- What are the base and height lengths of the triangles? (b = 6 in.; h = 8 in.)

Problem 4.2

During Problem 4.2 A, ask:

- How many blocks long is the base of Ashley’s model? (10)
- How many blocks wide is the base? (6)
- How can you find the number of blocks in the base layer? (Multiply 10 × 6.)

During Problem 4.2 B, ask:

- How can you find the volume, in cubic inches, of one of Brandon’s blocks? (Multiply \( \frac{1}{4} \times \frac{1}{4} \times \frac{1}{4} = \frac{1}{64} \).)
- Could Brandon have used Ashley’s blocks to make his model? (No; he couldn’t model \( \frac{3}{4} \) in. using only \( \frac{1}{2} \)-in. blocks.)

Summarize

To summarize the lesson, ask:

- What shapes are the faces of a cube? (6 squares)
- What faces does a square pyramid have? (1 square, 4 triangles)
- How can you find the surface area of a prism? (Find the area of each side of the prism and add the areas.)
- How can you find the volume of a rectangular prism? (Multiply its base times its height, or its length times its width times its height.)
- How is finding the volume of prisms with edge lengths that are fractions or mixed numbers like finding the volume of a prism with integer side lengths? (Use the same formulas to find the volume.)
Assignment Guide for Investigation 4

Problem 4.1, Exercises 1–15
Problem 4.2, Exercises 16–25

Answers to Investigation 4

Problem 4.1

A. 1. rectangular prism

2. Find the area of each rectangular piece of the net and then add the areas to find the total area of the net.

3. \[ A = (14 \times 4) + (14 \times 8) + (14 \times 4) + (14 \times 8) + (8 \times 4) + (8 \times 4) \]
\[ = 56 + 112 + 56 + 112 + 32 + 32 = 400 \text{ in.}^2 \]

4. The area of the net is equal to the surface area of the three-dimensional figure that can be made with the net.

B. 1. 2. Find the area of each piece of the net and then add the areas to find the total area of the net.

3. \[ A = (14 \times 8) + (14 \times 10) + (14 \times 6) + \frac{1}{2} (6) (8) + \frac{1}{2} (6) (8) \]
\[ = 112 + 140 + 84 + 24 + 24 = 384 \text{ in.}^2 \]

Problem 4.2

A. 1. 60 blocks

2. 600 blocks

3. 5 in. \times 3 in. \times 5 in.

4. \[ V = lwh = \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} = \frac{1}{8} \text{ in.}^3 \]

5. \[ V = lwh = 5 \times 3 \times 5 = 75 \text{ in.}^3 \]

6. The volume of a block that measures 1 in. on each side is 1 in.\(^3\), so one of Ashley’s blocks has a volume that is \( \frac{1}{8} \) that volume.

7. The volume of the model is the product of the number of blocks and the volume of each block: \[ 75 = 600 \times \frac{1}{8}. \]

8. The volume of a block that measures 1 in. on each side is 1 in.\(^3\), so one of Brandon’s blocks has a volume that is \( \frac{1}{64} \) that volume.

B. 1. \( \frac{1}{2} \div \frac{1}{4} = \frac{5}{2} \div \frac{1}{4} = \frac{5}{2} \times \frac{4}{1} = 10 \) blocks

wide; \( 3 \div \frac{1}{4} = 12 \) blocks long

2. \[ A = lw = \frac{1}{2} \times 3 = \frac{5}{2} \times 3 = \frac{15}{2} = 7 \frac{1}{2} \text{ in.}^2 \]

3. \[ \frac{2}{4} \div \frac{1}{4} = \frac{27}{4} \div \frac{1}{4} = \frac{27}{4} \times \frac{4}{1} = 27 \text{ blocks} \]

4. \( 10 \times 12 \times 27 = 3,240 \text{ blocks} \)

5. \[ V = Bh = \left( \frac{1}{4} \times \frac{1}{4} \right) \times \frac{1}{4} = \frac{1}{4} \times 4 \times 4 \]
\[ = \frac{1}{64} \text{ in.}^3 \]

6. \[ V = Bh = \frac{7}{2} \times 6 \frac{3}{4} = \frac{15}{2} \times 27 \frac{4}{2} = 15 \times 27 \]
\[ = 50 \frac{5}{8} \text{ in.}^3 \]

7. The volume of a block that measures 1 in. on each side is 1 in.\(^3\), so one of Brandon’s blocks has a volume that is \( \frac{1}{64} \) that volume.

8. The volume of the model is the product of the number of blocks and the volume of each block: \( 50 \frac{5}{8} = 3,240 \times \frac{1}{64} \).
Exercises
1. Sample (not to scale):

2. 6 rectangles
3. 2 triangles and 3 rectangles
4. 1 square and 4 triangles
5. Sample (not to scale):

6. Sample (not to scale):

7.

8.

9. 96 in.²
10. 184 m²
11. 243 cm²
12. 144 ft²
13. a. 4 ft × 6 ft, 4 ft × 6 ft, 12 ft × 6 ft, 12 ft × 6 ft, 12 ft × 4 ft, 12 ft × 4 ft
   b. 24 ft², 24 ft², 72 ft², 72 ft², 48 ft², 48 ft²
   c. 288 ft²
14. No; Surface Area = 15 × 12 + 15 × 12 + 12 × 8 + 12 × 8 + 15 × 8 + 15 × 8 = 792 in.²; 792 > 750.
15. a. square
   b. No, you also need the height of a triangular face.
16. 256 m³
17. 1,728 cm³
18. 294 in.³
19. 235\(\frac{1}{8}\) ft³
20. 1\(\frac{7}{8}\) in.³
21. 5\(\frac{5}{8}\) in.³
22. 13\(\frac{1}{8}\) in.³
23. a. \(\frac{1}{2}\)-inch blocks
   b. 280 blocks
   c. 35 in.³
24. A
25. a. 8 blocks
   b. \(\frac{3}{4}\) in. × 2\(\frac{1}{4}\) in. × 2 in.
   c. \(\frac{3}{8}\) in.³
   d. Possible answers: 1\(\frac{1}{2}\) in. × 1\(\frac{1}{2}\) in.; 1 in. × 2\(\frac{1}{4}\) in.; \(\frac{3}{4}\) in. × 3 in.; \(\frac{1}{2}\) in. × 4\(\frac{1}{2}\) in.; \(\frac{1}{4}\) in. × 9 in.
1. Seth needs to cover the box below with fabric so that it can be used as a prop in the school play. Seth has 375 in.$^2$ of fabric.

![Box Diagram]

a. Draw a net of the box. Name the shapes that make up the faces of the box.

b. Does Seth have enough fabric to cover this box? Explain how you know.

c. What is the longest rectangular box with a square base that measures 5 in. on each side that Seth could wrap with his fabric? Explain how you found your answer.

2. A tea company sells a flavored tea in the rectangular boxes shown below.

![Tea Boxes Diagram]

a. Without calculating, which box do you think holds the most tea? Explain your answer.

b. What is the volume of each box? How does this compare to your prediction?

c. Which box of tea would you expect to cost the least? Explain your reasoning.
Skill: Find the Surface Area

Find the surface area.

1. \( \text{5 cm} \times \text{4 cm} \times \text{3 cm} \times \text{14 cm} \)

2. \( \text{6 in.} \times \text{8 in.} \times \text{3 in.} \)

3. \( \text{2.5 m} \times \text{2.5 m} \times \text{2.5 m} \)

4. \( \text{5 cm} \times \text{12 cm} \times \text{20 cm} \)

Skill: Find the Volume

Find the volume.

7. \( \text{9 in.} \times \text{35 in.} \times \text{9 in.} \)

8. \( \text{1 1/2 in.} \times \text{1 1/2 in.} \times \text{1 1/2 in.} \)

9. \( \text{2 1/2 in.} \times \text{4 1/4 in.} \times \text{2 in.} \)

10. \( \text{3 3/4 yd} \times \text{7 1/2 yd} \times \text{1 3/4 yd} \)
1. Jimmy needs to wrap this rectangular box for a birthday present.

![Box Diagram]

a. Draw a net of the box and label the dimensions.

b. Jimmy has 600 in.$^2$ of wrapping paper. Is this enough for him to wrap the box? Explain how you found your answer.

2. Jimmy has more presents he needs to wrap. The other presents each come in its own box with the shape shown below.

![Triangular Prism Diagram]

a. Draw a net of the box and label the dimensions.

b. How many of these boxes could Jimmy wrap with 600 in.$^2$ of wrapping paper? Explain how you found your answer.
3. Latoya is going to build a rectangular storage shed for her backyard. The shed will have a volume of 432 ft³. Latoya sketched this net of the shed.

![Diagram of a rectangular storage shed with dimensions A: 9 ft, B: 6 ft, C: 6 ft, D: 6 ft, E: 6 ft, F: 6 ft.]

a. The base of the shed is the face labeled C. What is the height of the shed? Explain how you found your answer.

b. Latoya will paint the inside walls and ceiling of the shed, but not the floor. She finds cans of paint that will cover 48 ft² and cost $5.29 each. She also finds a gallon of paint that will cover 350 ft² and costs $29.99. Which should Latoya buy? Explain your answer.

4. Latoya finds plans for another rectangular storage shed that has dimensions of 10 ft × 5 ft × 7.5 ft. If she wants the shed with more storage space, which shed should she build? Explain your choice.
Investigation 4: Measurement

CC

A net is a two-dimensional model that can be folded into a three-dimensional figure. Prisms are three-dimensional figures that have two congruent and parallel faces that are polygons, such as rectangles or triangles. The rest of a prism’s faces are parallelograms. You can use nets of rectangular and triangular prisms to find their surface areas.

Problem 4.1

Ashley cuts nets from poster board and folds them to make three-dimensional models of buildings.

A. Ashley first cuts out the net shown at the right.
   1. What three-dimensional figure can she fold from this net?
   2. Ashley knows that she can use the formula \( A = lw \) to find the area of a rectangle, where \( l \) represents the rectangle’s length and \( w \) represents its width. Explain how to find the total area of the net.
   3. What is the area of the net?
   4. How is the area of the net related to the surface area of the prism?

B. Ashley wants to model the roof of a building. The triangular faces of the figure are parallel right triangles.
   1. Draw a net of the figure. Label the lengths of the sides.
   2. Explain how you can use the net to find the figure’s surface area.
   3. What is the surface area? Remember that the formula \( A = \frac{1}{2}bh \) is used to find the area of a triangle with a base \( b \) and a height \( h \).
Remember that you can find the volume of a rectangular prism by multiplying the area of its base by its height. Use one of these formulas:

\[ V = Bh, \text{ where } B \text{ is the area of the base, and } h \text{ is the height, or} \]

\[ V = lwh, \text{ where } l \text{ and } w \text{ are the length and width of the base, and } h \text{ is the height.} \]

**Problem 4.2**

Ashley and Brandon use various cubic blocks to model some of the buildings near their school.

**A.** Ashley’s blocks are \( \frac{1}{2} \) inch on each side. She starts her model by making a 5-inch by 3-inch rectangular base with the blocks.

1. How many blocks does she use?
2. Ashley adds 9 more layers to the base layer. How many blocks does she use in all?
3. What are the length \( (l) \), width \( (w) \), and height \( (h) \) of the model, in inches?
4. Use the formula \( V = lwh \) to find the volume, in cubic inches, of one of Ashley’s blocks. Show your work.
5. Use the formula \( V = lwh \) to find the volume, in cubic inches, of Ashley’s model. Show your work.
6. How is the volume of 1 block related to the volume of a block that measures 1 in. on each side?
7. Look at the volume of each block, the number of blocks used, and the volume of the model. How are these numbers related?
B. Brandon measures his blocks to be $\frac{1}{4}$ inch on each side. He models a building in the shape of a rectangular prism that is $2\frac{1}{2}$ in. wide, 3 in. long, and $6\frac{3}{4}$ in. high.

1. How many blocks wide and long is the base of Brandon’s model?
2. What is the area, in square inches, of the model’s base?
3. How many blocks tall is the model?
4. How many blocks did Brandon use for the model?
5. Use the formula $V = Bh$ to find the volume, in cubic inches, of one of Brandon’s blocks. Show your work.
6. Use the formula $V = Bh$ to find the volume, in cubic inches, of Brandon’s model. Show your work.
7. How is the volume of 1 block related to the volume of a block that measures 1 in. on each side?
8. Look at the volume of each block, the number of blocks used, and the volume of the model. How are these numbers related?

**Exercises**

1. Draw the three-dimensional figure modeled by this net.

![Net Diagram]

Name the two-dimensional shapes used to make the faces of each object.
Tell how many there are of each shape.

2. a rectangular prism, such as a shoebox
3. a triangular prism, like a tent
4. a pyramid, like the structures built by the Egyptians
Draw a net for each figure.

5. [Net of a triangular pyramid]

6. [Net of a triangular prism]

7. [Net of a rectangular prism]

8. [Net of a rectangular prism]

Find the surface area of each figure.

9. [Cube with dimensions 4 in. x 4 in. x 4 in.]

10. [Right rectangular prism with dimensions 5 m x 4 m x 6 m]

11. [Triangular prism with dimensions 9 cm x 9 cm x 9 cm]

12. [Rectangular prism with dimensions 6 ft x 6 ft x 3 ft]

13. A container has two rectangular ends that measure 4 ft by 6 ft, and another side that has a length of 12 ft.
   a. What are the measurements of each of the faces of the container?
   b. What are the areas of all of the faces of the container?
   c. What is the total surface area of the container?

14. Keira has 750 square inches of wrapping paper. Her package is shaped like a right rectangular prism that is 15 inches long, 12 inches wide, and 8 inches high. Does she have enough paper to cover her package? Explain.
15. The pyramid at the right has four faces that are congruent triangles.
   a. What shape is the base of the pyramid?
   b. If you know just the length of a side of the base, do you have enough information to find the surface area of the pyramid? Explain.

Find the volume of each rectangular prism.

16. 17.

18. 19.

For Exercises 20–22, each rectangular prism is built with $\frac{1}{2}$-in. blocks. Find the length, width, height, and volume of the prism.

20. 21. 22.
23. a. What size cubic blocks would you use to make a rectangular prism that is 2\(\frac{1}{2}\) in. by 3\(\frac{1}{2}\) in. by 4 in.? Explain your choice.

b. How many blocks would you need?

c. Give the volume of the model in cubic inches.

24. **Multiple Choice** Which set of dimensions describes the rectangular prism with the greatest volume?

   A. 3\(\frac{1}{2}\) in. by 2 in. by 5 in.

   B. 3 in. by 3 in. by 3\(\frac{1}{2}\) in.

   C. 4 in. by 2 in. by 4 in.

   D. 2\(\frac{1}{4}\) in. by 2 in. by 7 in.

25. Megan uses 216 cubic blocks to make a rectangular prism 9 blocks long and 3 blocks tall. Each block measures \(\frac{1}{4}\) inch on each side.

   a. How many blocks wide is the prism?

   b. What are the prism’s dimensions in inches?

   c. What is the prism’s volume, in cubic inches?

   d. Megan uses all of the bricks to make a new prism that is 6 bricks tall. Give 3 possible sets of dimensions of the base of the prism Megan could have made.
CC Investigation 5: Histograms and Box Plots

Mathematical Goals

- Display numerical data in histograms and box plots.
- Summarize numerical data sets by giving quantitative measures of center and variability.
- Summarize numerical data sets by describing any overall patterns and any striking deviations from an overall pattern, given the context in which the data were gathered.

Teaching Notes

In this investigation, students will consider that a data distribution may not have a definite center and that different ways to measure center yield different values. The median is the middle value. The mean is the value that each data point would take on if the total of the data values were redistributed equally.

Students also will explore how measures of variability (interquartile range or mean absolute deviation) can be useful for summarizing data. Two very different sets of data can have the same mean and median yet be distinguished by their variability. Students will learn to describe and summarize numerical data sets, identifying clusters, peaks, gaps, and symmetry, considering the context in which the data were collected.

Problem 5.1

Before Problem 5.1, review mean, median, and mode as measures of center. Ask:

- What is the mean value of a set of data? (the sum of the data values divided by the number of values)
- How would you find the mean number of points scored? (Add the scores and then divide by the number of scores, 22.)
- What is the median value of a set of data? (the middle number when the data values are ordered from least to greatest)
- If there are 7 values in a data set, which is the median? (The median is the 4th value, when the values are ordered from least to greatest.)
- How do you find the median value for a data set containing an even number of values? (Find the mean of the two middle values, when the values are ordered from least to greatest.)
- If there are 8 values in a data set, what is the median? (The median is the mean of the 4th and 5th values, when the values are ordered from least to greatest.)
- What are the modes of a set of data? (the values that appear most often)
During Problem 5.1 C, ask: How many values are in the lower half of the scores? (11)

After Problem 5.1, point out to students that box plots are good for comparing similar data very quickly, but do not show individual values. Ask:

• Can you tell from the box plot how many games the Panthers played? (No.)
• Can you tell from the box plot in how many games the Panthers scored more than 80 points? (No.)
• Can you tell from the box plot the fewest and most points scored? (Yes.)
• How is the median value of a data set affected if one data value lower than the median and one data value greater than the median are added to the set? (It does not change.)

**Problem 5.2**

Before Problem 5.2, introduce students to histograms, including some of their limitations. Point out the sample Homework histogram, and ask:

• What “frequency” does the vertical axis represent? (the number of students who spent that much time on homework)
• What does the bar above 10–19 indicate? (Six students spent between 10 and 19 minutes on homework.)
• How can you find the total number of students included in the data? (Add the numbers shown by the bars.)
• Does the histogram show what was the minimum amount of time any student spent on homework? (No, histograms do not display exact values.)
• Can you determine an exact median or mean from the histogram? (No, histograms do not display exact values.)

During Problem 5.2 A, ask:

• What information are you losing in transferring the data from the table to the histogram? (all of the individual values, including least value, greatest value, and the measures that come from the individual values, including mean, median, and mode)
• What benefits does the histogram offer? (It shows quickly that most students study 20–29 minutes, and the fewest study 40–49 minutes.)

**Problem 5.3**

During Problem 5.3, ask:

• What does the interquartile range represent? (the spread between the least and greatest values in the middle part of the data)
• How great would you characterize the variability of Grant's scoring? (Because the interquartile range is low, the data values do not vary too much; there is a relatively low variability in his scoring.)
**Problem 5.4**

*During Problem 5.4 B, ask: How do you find the distance between the data point and the mean?* (Find the absolute value of the difference of the values.)

*After Problem 5.4, ask:*

- *How great would you characterize the variability of the team’s scoring?* (Because the mean absolute deviation is low, most of the data values do not vary too much; there is a relatively low variability in the scoring.)
- *How are the data presented in this problem similar to the data presented in Problem 5.3?* (Both data sets show little variability, but have a value that is very different than the pattern of the rest of the values.)

**Summarize**

To summarize the lesson, ask:

- *What does the lower quartile of a set of data represent?* (the median of the lower half of the values)
- *What is the interquartile range?* (the difference between the upper and lower quartiles)
- *Which measure of center will be most affected if a single value much greater than the rest of the values is added to a data set?* (mean)
- *What is important to keep in mind about the intervals when making a histogram?* (They must be the same size, and there can be no gaps between them.)
- *What measures of center and variability cannot normally be found from data displayed only in a histogram?* (median, mean, mode, interquartile range, and mean absolute deviation)
- *What does a data set’s variability tell you about the data?* (the degree to which the data are spread out around an average value)
- *How do you find the mean absolute deviation of a data set?* (First, find the mean of the data set. Find the distance of each value in the set from that mean, and then find the mean of those distances.)

Students in the CMP2 program will further study standards 6.SP.4 and 6.SP.5.c in the Grade 7 Unit *Data Distributions.*

**Assignment Guide for Investigation 5**

- Problem 5.1, Exercises 1–13, 23
- Problem 5.2, Exercises 14–16
- Problem 5.3, Exercises 17–18, 22
- Problem 5.4, Exercises 19–21
Answers to Investigation 5

Problem 5.1

A. 68, 68, 68, 72, 73, 78, 80, 80, 82, 82, 85, 86, 86, 87, 88, 89, 90, 91, 91, 95, 96; minimum = 68; maximum = 96

B. 85.5; There is an even number of data values, so the median is the mean of the middle two values, 85 and 86; \((85 + 86) / 2 = 85.5\).

C. 78

D. 89

E. 1. \(89 - 78 = 11\)
   2. the difference between the maximum and minimum values of the middle half of the data

F. [Diagram showing box plot]

G. 1. to the left of the box showing the interquartile range
   2. The team most often scored more than 75 points in each game.

H. 1. It will increase the mean and median, but not affect the modes.
   2. \(\text{mean} = (68 + 68 + 68 + 72 + 73 + 78 + 80 + 80 + 82 + 82 + 85 + 86 + 86 + 87 + 88 + 89 + 90 + 91 + 91 + 95 + 96 + 96) / 23 = \approx 83.3; \text{median} = 86; \text{modes} = 68 \text{ and } 86\)
   3. The mean increased from \((68 + 68 + 68 + 72 + 73 + 78 + 80 + 80 + 82 + 82 + 85 + 86 + 86 + 87 + 88 + 89 + 90 + 91 + 91 + 95 + 96 + 96) / 22 = \approx 82.8 \text{ to } 83.3; \text{the median increased from } 85.5 \text{ to } 86; \text{the modes did not change.}\)

4. [Diagram showing box plot]

G. 1. to the left of the box showing the interquartile range
   2. The team most often scored more than 75 points in each game.

H. 1. It will increase the mean and median, but not affect the modes.
   2. \(\text{mean} = (68 + 68 + 68 + 72 + 73 + 78 + 80 + 80 + 82 + 82 + 85 + 86 + 86 + 87 + 88 + 89 + 90 + 91 + 91 + 95 + 96 + 96) / 23 = \approx 83.3; \text{median} = 86; \text{modes} = 68 \text{ and } 86\)
   3. The mean increased from \((68 + 68 + 68 + 72 + 73 + 78 + 80 + 80 + 82 + 82 + 85 + 86 + 86 + 87 + 88 + 89 + 90 + 91 + 91 + 95 + 96 + 96) / 22 = \approx 82.8 \text{ to } 83.3; \text{the median increased from } 85.5 \text{ to } 86; \text{the modes did not change.}\)

5. Box plots do not show a data set’s mean or mode.

6. The median increased by 0.5.

Problem 5.2

A. 1. 100; 51
   2. 51–60, 61–70, 71–80, 81–90, 91–100
   3. Five intervals divides the data into enough groups without making a histogram that is too large.

4. | Interval     | Frequency |
<table>
<thead>
<tr>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>51–60</td>
<td>2</td>
</tr>
<tr>
<td>61–70</td>
<td>11</td>
</tr>
<tr>
<td>71–80</td>
<td>10</td>
</tr>
<tr>
<td>81–90</td>
<td>6</td>
</tr>
<tr>
<td>91–100</td>
<td>3</td>
</tr>
</tbody>
</table>

5. [Bar chart showing distribution of winning scores]

6. Sample: Most of the winning scores were between 61 and 80 points, with very few scores of 60 or less.

B. 1. Sample:

2. Sample: Most of the scores were between 12 and 22 points, with very few scores of 11 points or less.

3. Sample: The intervals were different, with the scores in Part A being higher overall than the scores in Part B. The shapes of the data are similar. There are more data values shown in Part A than in Part B.
Problem 5.3
A. 14; 12; 16
B. 16 – 12 = 4; The interquartile range is low, so Grant’s scoring was quite consistent throughout the season.
C. Most of the data fell within a very narrow range, except for the low value of 0.
D. Yes; the number 0 falls outside the pattern of bunched data within the interquartile range. That value is much less than most of the rest of the numbers.

Problem 5.4
A. 9; (7 + 6 + 17 + 8 + 7 + 9) ÷ 6 = 54 ÷ 6 = 9
B. 7: 2; 6 : 3; 17: 8; 8: 1; 7: 2; 9: 0
C. 1. 2 + 3 + 8 + 1 + 2 + 0 = 16
2. 16 ÷ 6 = 2.67
D. The number of goals scored is, on average, about 2.67 goals less than or about 2.67 goals greater than the mean.
E. Yes; the score of 17 is much higher than the rest of the scores.

Exercises
1. 3, 4, 9, 12, 18;
2. 15, 18, 25, 29, 32;
3. 2.9, 4, 6.2, 8.05, 9.3;
4. a. 7.9, 9.1, 11, 13.2, 14;
   b. 
   c. Yes; the box plots show that this year’s seedlings are smaller than last year’s seedlings. All of the five-number summary values are less.
5. a. Box plots summarize key values that show distributions of data sets, and by making two box plots together, differences in these values can be seen easily.
   b. Box plots do not show the mean, and do not show the individual data values from which the mean could be calculated.
6. The mean increases from 84.2 to 91; the median increases from 88 to 90; there is no mode for either data set.
7. The score increased from 8 to 8.5.
8. a. First 8 Games
   b. The mean increased from 4.5 to 5.8; the median increased from 5 to 6; the mode increased from 5 to 8.
9. The mean increased from 5 m to 6.5 m; the median increased from 5.5 m to 6 m; the modes changed from 0 and 6 to just 6.
10. The mean increased the most, by 1.5 m.
11. First Round
    Bonus Round
12. a. 

\[ \text{First 8 Flights} \]

\[ \text{Second 8 Flights} \]

b. The mean increased from 20.1 ft to 43.5 ft; the median increased from 20 ft to 45 ft; the mode increased from 20 ft to 47 ft.

13. Sample:

\[ \text{Ages of Mall Shoppers} \]

\[ \begin{array}{c|c|c|c}
\text{Age} & \text{Frequency} \\
10–19 & 2 \\
20–29 & 8 \\
30–39 & 6 \\
40–49 & 4 \\
\end{array} \]

14. Sample:

\[ \text{History Exam Class Grades} \]

\[ \begin{array}{c|c|c|c|c}
\text{Grade} & \text{Frequency} \\
66–72 & 8 \\
73–79 & 6 \\
80–86 & 4 \\
87–93 & 2 \\
94–100 & 0 \\
\end{array} \]

15. Sample:

\[ \text{Pumpkin Weights} \]

\[ \begin{array}{c|c|c|c|c}
\text{Weight (in lb)} & \text{Frequency} \\
0–6 & 2 \\
7–13 & 4 \\
14–20 & 6 \\
21–27 & 2 \\
\end{array} \]

16. B

17. a. Half of the data are clustered within a small interquartile range, between 87 and 91.

b. The value 76 is much less than most of the rest of the data values, and far from the median.

18. about 3.2

19. about 2.2

20. about 2.9

21. The interquartile range is very small, 3, and all but one value in the set falls within 5 of the median value, 12. The value 45 is much greater than any other value in the data set.

22. a. His number of points increased after the first 8 games.

b. Brandon spent more time practicing taking 3-point shots, so he was more successful in making them.

c. The median is the only measure of these that is shown on a box plot, so that is the only measure that will be shown between box plots of different data sets.

d. First 8 Games

\[ \text{All 20 Games} \]

The box plots show that the median increased from 1.5 in the first 8 games to 6 for all 20 games. The quartiles and maximum values also increased, as did the interquartile range.
1. Paige recorded the numbers of states that her classmates had visited.

<table>
<thead>
<tr>
<th>Numbers of States Visited</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
</tr>
<tr>
<td>2</td>
</tr>
<tr>
<td>2</td>
</tr>
<tr>
<td>0</td>
</tr>
</tbody>
</table>

a. What are the mean, median, and mode numbers of states visited?

b. Make a box plot to display the data.

c. What is the interquartile range? What does that number tell you about how spread out the data are?

d. Describe the overall pattern of the data.

e. What is the distance of each data value from the mean?

f. What is the mean absolute deviation of the data? Show your work.

g. What does the mean absolute deviation tell you about how spread out the data are?

2. The table shows the ages of a random sample of spectators at a hockey game.

<table>
<thead>
<tr>
<th>8</th>
<th>14</th>
<th>22</th>
<th>15</th>
<th>21</th>
<th>7</th>
<th>31</th>
</tr>
</thead>
<tbody>
<tr>
<td>17</td>
<td>38</td>
<td>6</td>
<td>42</td>
<td>14</td>
<td>30</td>
<td>15</td>
</tr>
<tr>
<td>7</td>
<td>12</td>
<td>21</td>
<td>12</td>
<td>9</td>
<td>45</td>
<td>10</td>
</tr>
<tr>
<td>40</td>
<td>36</td>
<td>7</td>
<td>17</td>
<td>25</td>
<td>6</td>
<td>45</td>
</tr>
</tbody>
</table>

a. What are the least and greatest ages?

b. Divide the range of the data into equal intervals to be represented by bars on a histogram. Give the range for each interval and explain why you chose that number of intervals.

c. Make a histogram of the data.

d. Summarize what the histogram shows about the data.
Skill: Finding Mean Absolute Deviation

Find the mean absolute deviation for the set of data.

1. 13 18 17 19 11 15
2. 10 8 7 8 11 9
3. 70 72 83 88 93 81 92
4. 2 13 14 3 9 20 11
5. 39 31 37 39 34 35 34 30
6. 105 111 124 120 118 121 92 110

Skill: Reading Box Plots

Give the five-number summary for each box plot. Find the range and the IQR.

7.

8.

9.

10.
Check-Up

1. Mindy recorded in the table the height of each player on the basketball team.
   a. What are the mean, median, and mode heights?

   b. Make a box plot to display the data.

   c. What does the interquartile range show about how consistent the players' heights are?

   d. Describe the overall pattern of the data, and identify any height that does not follow that pattern. Explain what about the value makes it unusual.

   e. What is the distance of each data value from the mean?

   f. What is the mean absolute deviation of the data? Show your work.

   g. What does the mean absolute deviation tell you about the heights of the basketball players?

   h. After Mindy made the box plot, the player who is 57 in. tall left the team. How does this change affect the mean, median, and mode of the heights?
2. The table shows the heights, in inches, of the basketball players on a USA Men’s National Team.

<table>
<thead>
<tr>
<th>83</th>
<th>80</th>
<th>75</th>
<th>81</th>
<th>82</th>
<th>78</th>
<th>85</th>
</tr>
</thead>
<tbody>
<tr>
<td>75</td>
<td>81</td>
<td>78</td>
<td>80</td>
<td>75</td>
<td>80</td>
<td>81</td>
</tr>
<tr>
<td>83</td>
<td>78</td>
<td>80</td>
<td>82</td>
<td>81</td>
<td>84</td>
<td>84</td>
</tr>
<tr>
<td>82</td>
<td>76</td>
<td>84</td>
<td>82</td>
<td>72</td>
<td>82</td>
<td>81</td>
</tr>
<tr>
<td>73</td>
<td>75</td>
<td>82</td>
<td>76</td>
<td>79</td>
<td>75</td>
<td>75</td>
</tr>
</tbody>
</table>

a. Make a histogram of the data.

b. Summarize what the histogram shows about the data.

3. The table shows the heights, in inches, of the basketball players on a USA Women’s National Team.

<table>
<thead>
<tr>
<th>73</th>
<th>71</th>
<th>69</th>
<th>73</th>
<th>72</th>
</tr>
</thead>
<tbody>
<tr>
<td>75</td>
<td>74</td>
<td>73</td>
<td>78</td>
<td>68</td>
</tr>
<tr>
<td>75</td>
<td>69</td>
<td>73</td>
<td>67</td>
<td>72</td>
</tr>
<tr>
<td>76</td>
<td>69</td>
<td>72</td>
<td>69</td>
<td>71</td>
</tr>
</tbody>
</table>

a. Make a histogram of the data.

b. Summarize what the histogram shows about the data.

c. Compare the histogram to the one you made in Problem 3. Explain what the differences in the graphs illustrate about the data.
A box plot is constructed from the **five-number summary**: the minimum value, lower quartile, median, upper quartile, and maximum value.

The Panthers scored the following numbers of points during games this season:

68 91 86 89 88 82 95 85 80 78 82
68 86 96 73 68 91 80 90 86 72 87

**A.** Order the set of data from the least to the greatest. What are the minimum and maximum values?

**B.** Find the median. Explain how you found this value.

**C.** The **lower quartile** is the median of the lower half of the scores. What is the lower quartile of the data?

**D.** The **upper quartile** is the median of the upper half of the scores. What is the upper quartile of the data?

**E.** 1. Find the difference between the upper quartile and the lower quartile. This difference is called the **interquartile range**.
   2. What does the interquartile range represent?

**F.** 1. Draw a number line from 60 to 100.
   2. Above your number line, draw vertical line segments at the values you found for the median, the lower quartile, and the upper quartile.
   3. Connect the vertical lines to form a rectangle.
   4. Locate the value you identified as the minimum and draw a line to the left from the rectangle to meet that point.
   5. Locate the value you identified as the maximum and draw a line to the right from the rectangle to meet that point.
G. 1. Where is a score of 75 found on your box plot?
2. What does the location of 75 tell you about the performance of the team this season?

H. The Panthers scored 96 points in the first game of the playoffs.
1. How do you think this change will affect the mean, median, and mode of the numbers of points scored?
2. Find the mean, median, and mode for the 23 games played this season.
3. How did the mean, median, and mode change?
4. Make a new box plot to include the data for all 23 games.
5. Can you see the changes to the mean, median, or mode between the first and second box plots? Explain why or why not.
6. Which of the five-number-summary values changed the most? Explain your answer.
A histogram is a type of bar graph in which the bars represent numerical intervals. Each interval must be the same size, and there can be no gaps between them. In this histogram, there are 5 equal intervals of 10 minutes each.

### Problem 5.2

**A.** The table shows the winning scores in the first round of the basketball tournament.

<table>
<thead>
<tr>
<th>Score</th>
<th>Frequency</th>
</tr>
</thead>
<tbody>
<tr>
<td>90</td>
<td>6</td>
</tr>
<tr>
<td>79</td>
<td>6</td>
</tr>
<tr>
<td>100</td>
<td>1</td>
</tr>
<tr>
<td>73</td>
<td>1</td>
</tr>
<tr>
<td>69</td>
<td>1</td>
</tr>
<tr>
<td>70</td>
<td>1</td>
</tr>
<tr>
<td>89</td>
<td>1</td>
</tr>
<tr>
<td>62</td>
<td>1</td>
</tr>
<tr>
<td>97</td>
<td>1</td>
</tr>
<tr>
<td>64</td>
<td>1</td>
</tr>
<tr>
<td>68</td>
<td>1</td>
</tr>
</tbody>
</table>

1. What are the greatest and least winning scores?
2. Divide the range of the data into equal intervals that will be represented by bars on the histogram. Give the range for each interval.
3. Explain why you chose that number of intervals.
4. Make a table to show the frequency of scores in each interval.
5. Make a histogram of the data. Draw a bar for each interval to represent the frequency.
6. Summarize what the histogram shows about the data.

**B.** The table shows the scores of all of the games in the football playoffs.

<table>
<thead>
<tr>
<th>Score</th>
<th>Frequency</th>
</tr>
</thead>
<tbody>
<tr>
<td>33</td>
<td>1</td>
</tr>
<tr>
<td>30</td>
<td>1</td>
</tr>
<tr>
<td>14</td>
<td>1</td>
</tr>
<tr>
<td>24</td>
<td>1</td>
</tr>
<tr>
<td>51</td>
<td>1</td>
</tr>
<tr>
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<td>34</td>
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</tr>
<tr>
<td>34</td>
<td>1</td>
</tr>
<tr>
<td>34</td>
<td>1</td>
</tr>
</tbody>
</table>

1. Make a histogram of the data.
2. Summarize what the histogram shows about the data.
3. Compare the histogram to the one you made in Part A. Explain the differences the graphs illustrate about the data.
You can summarize data sets using measures of variability. **Variability** is the degree to which data are spread out around a center value.

**Problem 5.3**

The box plot shows the number of points Grant scored in each game.

![Box Plot](image)

A. What are the median, lower quartile, and upper quartile of the data?

B. What is the interquartile range? What does that number tell you about the how consistent Grant’s scoring was this season?

C. Describe the overall pattern of the data.

D. Do there appear to be any scores that do not follow the pattern of the rest of the data? Explain what those values represents and what makes them unusual.

A data set’s mean absolute deviation is the average distance of all data values from the mean of the set. First, find the mean of the data set. Then find the distance of each value in the set from that mean and find the average of those distances.

**Problem 5.4**

Paige’s lacrosse team scored the following numbers of goals in the first six games of the season: 7, 6, 17, 8, 7, 9.

A. What is the mean number of goals scored? Show your work.

B. What is the distance of each data value from the mean?

C. 1. What is the total distance of all of the data points from the mean?
   2. The mean absolute deviation is the average of the these distances. What is the mean absolute deviation of the data?

D. What does the mean absolute deviation tell you about the numbers of goals scored?

E. Do you notice any value that does not follow the pattern of the rest of the data? Explain what makes that value unusual.
Exercises

For Exercises 1–3, use a five-number summary to draw a box plot for each set of data.

1. 12 7 3 11 13 18 8 4 3 10
2. 26 16 25 30 29 21 18 32 25 15 20
3. 4.2 3.8 6.2 7.8 8.3 2.9 6.8 9.3 4.3

4. A farmer starts 9 tomato plants in a greenhouse several weeks before spring. The seedlings look a little small this year so the farmer decides to compare this year’s growth with last year’s growth.

This year’s growth is measured in inches as:
12 8.4 10 9.8 14 7.9 11 12.7 13.7

a. Use a five-number summary to draw a box plot for this set of data. Mark your number line from 0 to 20.

b. Last year, the five-number summary for the tomato plants was 9, 11, 13.4, 16, 17. Draw a box plot for this set of data. Mark your number line from 0 to 20.

c. Write this year’s summary above last year’s summary. Is the farmer’s concern justified? Why or why not?

5. a. Explain why you would use a box plot when you have similar data to compare.

b. Explain why you would not use a box plot if you needed to show the mean of the data.

6. CJ scored 85, 88, 94, 90, and 64 on math tests so far this grading period. His teacher allows students to retake the test with the lowest score and substitute the new test score. CJ scores a 98 on the retest. How does substituting the new test score affect the mean, median, and mode?

7. During a dance competition, Laura’s dance team received scores of 9, 9, 8, 9, 10, 8, 3, and 8 from the judges. For each team, the highest and lowest scores are removed. The remaining scores are then averaged to find the team’s final score. How was the team’s final score affected when the highest and lowest scores were removed?

8. a. During the first 8 games of the basketball season, Rita made the following number of free throws: 0, 3, 5, 5, 4, 8, 5, and 6. During the next 7 games she made 8, 8, 7, 9, 8, and 3 free throws. Make a box plot showing the data for the first 8 games and then the data for all of the games.

b. How did the mean, median, and mode of the free-throw data change from the first 8 games compared to all of the games?
For Exercises 9–11, use the information below.

Teams of two competed in the egg-toss distance competition. If the egg breaks, the distance is 0. The results for Dave and Paul’s team are shown below for the first round.

<table>
<thead>
<tr>
<th>Toss</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
</tr>
</thead>
<tbody>
<tr>
<td>Distance (in meters)</td>
<td>3.5</td>
<td>0</td>
<td>6</td>
<td>6</td>
<td>0</td>
<td>5</td>
<td>8.5</td>
<td>11</td>
</tr>
</tbody>
</table>

In a bonus round, each team can replace 1 toss from the first round. Dave and Paul make a toss of 12 meters.

9. How did the mean, median, and mode change after the toss from the bonus round?

10. Which measure—mean, median, or mode—changed the most?

11. Make a box plot using the data after the first round and then using the data after the bonus round.

12. Faye is writing an article in the school newspaper about the school’s paper airplane flying competition. She records Wheeler’s first flights in the table below.

<table>
<thead>
<tr>
<th>Wheeler’s Paper Airplane Competition Results</th>
</tr>
</thead>
<tbody>
<tr>
<td>Flight</td>
</tr>
<tr>
<td>Distance (in feet)</td>
</tr>
</tbody>
</table>

After the first 8 flights, Wheeler adds a paper clip to the nose of his airplane. Faye records the results of his next 8 flights in the table below.

<table>
<thead>
<tr>
<th>Wheeler’s Paper Airplane Competition Results</th>
</tr>
</thead>
<tbody>
<tr>
<td>Flight</td>
</tr>
<tr>
<td>Distance (in feet)</td>
</tr>
</tbody>
</table>

a. Make a box plot showing the data for the first 8 flights and then the data for the second 8 flights.

b. How did the mean, median, and mode of the flight distances change from the first 8 flights to the second 8 flights?
For Exercises 13–15, make a histogram to display the set of data.

13. ages of mall shoppers:
   23  33  21  18  17  45  40  23
   12  31  27  27  29  24  14  40
   19  18  25  17  36  40  38  20

14. class grades on a history exam:
   97  84  93  76  87  100  92  90
   70  85  83  99  90  89  84  91
   100  96  76  74  73  87  80  93

15. weights of pumpkins (in lb):
   5   16  23  8   7  9  12  15
   20  15  7  18  6  6 21  16
   8   11  12  16 10 20 23  9
   14  24  17  7  6 18  9 10

16. Multiple Choice  What is the interquartile range of the data?
   A. 8
   B. 20
   C. 10
   D. 24

17. a. Describe the overall pattern of the data shown in the box plot.

b. Identify any data value that is far outside the pattern and explain why it is outside the pattern.
For Exercises 18–20, find the mean absolute deviation for the set of data.

18. 21 23 18 27 30 24
19. 88 89 86 89 90 82
20. 2.4 2.8 2.1 2.7 13.0 2.5

21. Describe the overall pattern of data in the following set. Identify any data value that is far outside the pattern and explain why it is outside the pattern.
11 13 9 12 14 12 10 12
7 9 13 11 12 10 45 13

22. After the eighth game and for the rest of the season, Brandon spent an hour after each practice working on 3-point baskets. He also practiced for another half-hour when he got home. His 3-point basket data for the entire season are shown below.

<table>
<thead>
<tr>
<th>Game</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
<th>9</th>
<th>10</th>
</tr>
</thead>
<tbody>
<tr>
<td>Points</td>
<td>0</td>
<td>3</td>
<td>0</td>
<td>0</td>
<td>6</td>
<td>3</td>
<td>3</td>
<td>0</td>
<td>6</td>
<td>6</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Game</th>
<th>11</th>
<th>12</th>
<th>13</th>
<th>14</th>
<th>15</th>
<th>16</th>
<th>17</th>
<th>18</th>
<th>19</th>
<th>20</th>
</tr>
</thead>
<tbody>
<tr>
<td>Points</td>
<td>12</td>
<td>12</td>
<td>12</td>
<td>15</td>
<td>6</td>
<td>12</td>
<td>15</td>
<td>9</td>
<td>12</td>
<td>12</td>
</tr>
</tbody>
</table>

a. How did the data change after the first 8 games, if at all?
b. Why do you think the data did, or did not, change? Explain.
c. When data in a data set change, changes in which measures (mean, median, mode) will be shown in box plots? Explain your thinking.
d. Display the data from Brandon’s first 8 games in a box plot. Display the data from all 20 games in another box plot. Explain how differences between the two groups of data are shown in the plots.
Investigation 1 Additional Practice

1. a. Calvin’s sister would prefer the recipe with the lowest ratio of nuts rather than the lowest amount of nuts. Finding the ratio of nuts to granola for each recipe:

   A: \( \frac{8}{4} = 2 \), B: \( \frac{6}{6} = 1 \), C: \( \frac{7}{8} \), and D: \( \frac{12}{8} = \frac{3}{2} \).

   The lowest ratio is recipe C, so Calvin’s sister might prefer that.

   b. Recipe D; Calvin starts with 6 tablespoons of nuts. Recipe A has a nut-granola ratio of 2:1, and recipe D has a ratio of 3:2. For 6 tablespoons of granola, recipe A calls for 12 tablespoons of nuts, so Calvin would need to add 12 – 6 = 6 more tablespoons of nuts. For 6 tablespoons of granola, recipe D calls for 9 tablespoons of nuts, so Calvin would need to add 9 – 6 = 3 more tablespoons of nuts. 3 < 6, so Calvin would reach the nut-granola ratio of recipe D first.

2. a. Distances Driven

<table>
<thead>
<tr>
<th>Time (hours)</th>
<th>Distance (in mi)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>65</td>
</tr>
<tr>
<td>2</td>
<td>130</td>
</tr>
<tr>
<td>3</td>
<td>195</td>
</tr>
<tr>
<td>4</td>
<td>260</td>
</tr>
</tbody>
</table>

b. 5 days; Convert the distances to miles:

   Ottawa to Montreal: \( 200 \times 0.62 \approx 124 \text{ mi} \);
   Montreal to Quebec: \( 253 \times 0.62 \approx 160 \text{ mi} \);
   Quebec to Halifax: \( 1,022 \times 0.62 \approx 630 \text{ mi} \).

   At a speed of 65 mph, Ottawa to Montreal would take \( 124 \div 65 \approx 1.9 \text{ h} \), which is 1 day’s drive. Montreal to Quebec would take \( 160 \div 65 \approx 2.5 \text{ h} \), which is 1 day’s drive. Quebec to Halifax would take \( 630 \div 65 \approx 10 \text{ h} \), which is 3 days’ drive. \( 1 + 1 + 3 = 5 \text{ days} \)

Skill: Write Equivalent Fractions

1. \( x = 6 \)
2. \( x = 3 \)
3. \( x = 14 \)
4. \( x = 10 \)
5. \( x = 3 \)
6. \( x = 1 \)
7. \( x = 6 \)
8. \( x = 40 \)
9. \( x = 12 \)
10. \( x = 6 \)

Skill: Find the Unit Rate

11. 0.5
12. 0.2
13. 6
14. about 0.67
15. 0.6
16. 1.75
17. 1.6
18. about 0.42
19. about 0.29
20. 1.5
21. $2.49/lb
22. 52.5 mi/h
Investigation 1 Check-Up
1. a. Quinn; Compare the unit rates in minutes per bracelet: Zack: 30 ÷ 3 = 10 min/bracelet; Tine: 45 ÷ 5 = 9 min/bracelet; Ernie: 28 ÷ 4 = 7 min/bracelet; Quinn: 36 ÷ 6 = 6 min/bracelet; 6 < 7 < 9 < 10.
b. 16 bracelets; Zack can make 60 min ÷ 10 min/bracelet = 6 bracelets in 1 hour. Quinn can make 60 min ÷ 6 min/bracelet = 10 bracelets in 1 hour. 6 + 10 = 16 bracelets.
2. a. Lance; Raul; Convert speeds to mi/h:
Marlon: 44.8 km/h ÷ 1.6 km/mi ≈ 28 mi/h;
Bernhard: 44 km/h ÷ 1.6 km/mi ≈ 27.5 mi/h. Order the speeds: 28.5 > 27.5 > 27.
b. (26 + 28.5 + 27) ÷ 3 = 81.5 ÷ 3 = 27.17 mi/h; 27.17 mi/h × 1.6 km/mi ≈ 43.5 km/h
3. a. Chocolate chunk: $2.40 ÷ 12 cookies = $0.20 per cookie; peanut butter: $2.64 ÷ 12 cookies = $0.22 per cookie; sugar: $2.88 ÷ 12 cookies = $0.24 per cookie
b. Chocolate chunk: profit = (18 × $0.75) – (18 × $0.20) = $9.90; peanut butter: profit = (12 × $0.75) – (12 × $0.22) = $6.36; sugar: profit = (16 × $0.75) – (16 × $0.24) = $8.16. Chocolate chunk generates the most profit per hour.
c. Chocolate chunk: 6 – $1\frac{1}{2} = 4\frac{1}{2}$ hours of sales; $4\frac{1}{2} \times $9.90 = $44.55.$
Peanut butter: 6 – 1 = 5 hours of sales; 5 × $6.36 = $31.80.$
Sugar: 6 – $3\frac{3}{4} = 5\frac{1}{4}$ hours of sales;
$5\frac{1}{4} \times $8.16 = $42.84. Chocolate chunk cookies would generate the greatest profit over 6 hours.
d. Yes; Cost per cookie = $2.52 ÷ 12 = $0.21. Profit per hour = (22 × $0.75) – (22 × $0.21) = $11.88. Over 6 hours: 6 – 2 = 4 hours of sales; 4 × $11.88 = $47.52 profit. Oatmeal raisin would be more profitable than any other type of cookie.
Investigation 2 Additional Practice
1. a. the length of the rental period  
   b. $t$  
   c. Bob’s: $3t + 9$; Cycle Center: $5t$  
   d. | Hours | Bob’s | Cycle Center |
      |------|------|-------------|
      | 1    | $12$ | $5$         |
      | 2    | $15$ | $10$        |
      | 3    | $18$ | $15$        |
      | 4    | $21$ | $20$        |
      | 5    | $24$ | $25$        |
      | 6    | $27$ | $30$        |

e. Bob’s Bike Rentals; For 4 hours, Cycle Center is cheaper, but for 5 or 6 hours, Bob’s is cheaper, so for the greater part of that 4–6 hour period, Bob’s is a better deal.

2. a. $(3.75 \times 6) + (2.25 \times 6) = 22.5 + 13.5 = $36  
   b. $6(3.75 + 2.25) = 6(6) = $36  
   c. The costs are the same, so the expressions are equivalent.  
   d. Reece’s method was easier because adding the prices gave a whole number to multiply, which was easier than multiplying with decimals twice.

Skill: Work Backward
8. 21 songs; $26.30 - $20 = $6.30; $6.30 \div $0.30 = 21  
9. 17 songs; $10.20 \div 2 = $5.10/mo; $5.10 \div $0.30 = 17  
10. 32 songs; $6.60 \div $0.30 = 22; 22 + 10 = 32  
11. 6 songs; $6 - $4.20 = $1.80; $1.80 \div $0.30 = 6

Investigation 2 Check-Up
1. a. $12h$, where $h$ represents the number of sections in his History workbook  
   b. $12(8) = 96$ min, or 1.6 h  
   c. $12(h + p)$  
   d. $12h + 12p$; The distributive property can be used to show that $12(h + p) = 12h + 12p$.  
   e. $12(7 + 11) = 12(18) = 216$ min, or 3.6 h
2. a. $17 + d$; 19, 22, 27, 42  
   b. $14 + 17 = 17 + d$; The commutative property of addition states that $a + b = b + a$, so $d = 14$, and Ella has 14 dimes.
3. a. $4(4.00) + 9(2.50)$  
   b. $4(4.00) + 9(2.50) + 6(3.00) = 16.00 + 22.50 + 18.00 = $56.50  
   c. $(20 \div 4) + (15 \div 2.5) = 5 + 6 = 11$ packages  
   d. $3(2 \cdot 2.50)$, or $3(5.00)$  
   e. No; $56.50 + 3(5.00) = 56.50 + 15.00 = 71.50$ per group; $71.50 \times 6 = 429$; $429 > $400.

Skill: Write and Evaluate Expressions
1. $36 \div n$  
2. $3 \times p$  
3. $s - 24$  
4. $b + 6$  
5. $4(9) - 7 = 36 - 7 = 29$  
6. $54 - 3(8) = 54 - 24 = 30$  
7. $\frac{64}{4} \div 8 = 16 \div 8 = 2$
CC Investigation 3 Answers to Additional Practice, Skill Practice, and Check-Up

Investigation 3 Additional Practice
1. a. Tuesday and Wednesday
   b. Wednesday to Thursday
   c. Monday: |+4| = 4; Tuesday: |+2| = 2; Wednesday: |−2| = 2; Thursday: |0| = 0; Friday: |−5| = 5; Thursday, Tuesday/Wednesday, Monday, Friday

2. a. A(2, 3); B(−2, 3); C(−2, 0)
   b. 1 unit; point B to A = 4 units; point B to C = 3 units; 4 − 3 = 1
   c. 14 units; P = 3 + 4 + 3 + 4 = 14

3. a. h ≥ 48; h represents the height in inches to be able to ride the new ride
   b. The closed circle indicates that 48 is part of the solution.
   c. An infinite number; the arrow points to the right, indicating there is no end to the number of solutions.
   d. Yes, for example the solution h = 500 does not make sense because no one is 500 inches tall.

Skill: Identify Ordered Pairs
1. (1, 3)
2. (−2, 2)
3. (1, 0)
4. (3, −1)
5. (1, −2)
6. (0, −3)
7. (0, 0)
8. (−3, 0)
9. (−3, −2)
10. (−3, 3)
11. (3, 2)
12. (0, 3)

Skill: Interpret Graphs of Inequalities
13. x > 4
14. x ≤ 36
15. x < −2
16. x ≥ −60
17. x ≤ 13.5
18. x > 30

Investigation 3 Check-Up
1. a. −2
   b. yellow; green; red
   c. yellow and blue
   d. yellow; blue

From biggest loss to biggest gain, the stocks are C, D, A, and B.
   b. Find the absolute value of each change, and order them from least to greatest:
      |+2.45| = 2.45; |+7.16| = 7.16; |−8.32| = 8.32; |−0.98| = 0.98; 0.98 < 2.45 < 7.16 < 8.32, so the stocks in order from least to greatest price change are D, A, B, and C.

3. a. Points A and B have the same y-coordinate, 3, but different x-coordinates. Points B and D have the same x-coordinate, but different y-coordinates. Points A and B have coordinates that are opposite integers.
   b. 180° rotation around the origin, or reflection across the x-axis and a reflection of the image across the y-axis
   c. 14 units; The other corners of the rectangle would be located at (−4, 3) and (1, 1); the side lengths are 5, 5, 2, and 2.

4. a. m ≥ 60
   b. The graph shows that any amounts greater than or equal to $60 are a solution to the inequality.
   c. 14 units; The other corners of the rectangle would be located at (−4, 3) and (1, 1); the side lengths are 5, 5, 2, and 2.
Investigation 4 Additional Practice
1. a. There are 2 squares and 4 other congruent rectangles.
   b. No; \( SA = 2(5 \times 5) + 4(18 \times 5) = 2(25) + 4(90) = 50 + 360 = 410 \text{ in.}^2 \);  
   \( 410 > 375 \)
   c. 16.25 in.; \( SA = 2(5 \times 5) + 4(h \times 5); 375 = 50 + 20h; 20h = 325; h = 16.25 \)

2. a. Answers will vary.
   b. \( V_1 = lwh = 6 \times 6 \times 12 = 432 \text{ cm}^3 \);
      \( V_2 = lwh = 8 \times 8 \times 7.5 = 480 \text{ cm}^3 \);
      \( V_3 = lwh = 10.2 \times 8 \times 5 = 408 \text{ cm}^3 \);
      Comparisons to predictions will vary.
   c. The third box holds the least amount of tea, so it should cost the least.

Skill: Find the Volume
5. 2,835 in.\(^3\)
6. \( \frac{1}{8} \text{ in.}^3 \)
7. 21\(\frac{1}{4} \text{ in.}^3 \)
8. 49\(\frac{7}{32} \text{ yd}^3 \)

Investigation 4 Check-Up
1. a. 15 in.
   14 in.
   3 in.
   3 in.

Skill: Find the Surface Area
1. 180 cm\(^2\)
2. 180 in.\(^2\)
3. 37.5 m\(^2\)
4. 432 cm\(^2\)

b. Yes; \( SA = 2(15 \times 14) + 2(15 \times 3) + 2(14 \times 3) = 420 + 90 + 84 = 594 \text{ in.}^2 \);
   \( 594 < 600 \).
2. a. 3 boxes; $SA = (10 \times 6) + (8 \times 6) + (6 \times 6) + 2 \left( \frac{1}{2} \times 6 \times 8 \right) = 60 + 48 + 36 + 48 = 192$; $600 \div 192 = 3.1$

3. a. 8 ft; $V = lwh$; $432 = 9 \times 6 \times h$

\[ 432 = 54h; \quad h = 8 \]

b. $SA = 2(9 \times 8) + 2(8 \times 6) + (9 \times 6) = 144 + 96 + 54 = 294 \text{ ft}^2$. $294 \div 48 = 6.125$, so she would need 7 cans at a total cost of $7 \times 5.29 = 37.03$. One gallon would cover the area: $350 > 294$, at a total cost of $29.99$. It is cheaper to buy one gallon.

4. $V = lwh = 10 \times 5 \times 7.5 = 375 \text{ ft}^3$; $432 > 375$, so she should build the shed in Exercise 3 because it holds more.
Investigation 5 Additional Practice

1. a. mean = 2.2; median = 1.5; modes = 0 and 1
   b. ![Bar graph showing distribution]
   c. IQR = 4 – 0.5 = 3.5; the data are fairly spread out.
   d. Most of the values are between 0 and 4, with a few greater than 4.
   e. 0 values: 2.2; 1 values: 1.2; 2 values: 0.2; 3 values: 0.8; 4 values: 1.8; 5 values: 2.8; 6 values: 3.8
   f. MAD = 1.76; MAD = \(\frac{(5 \times 2.2) + (5 \times 1.2) + (3 \times 0.2) + (1 \times 0.8) + (2 \times 1.8) + (2 \times 2.8) + (2 \times 3.8)}{20} \)
      \(= \frac{(11 + 6 + 0.6 + 0.8 + 3.6 + 5.6 + 7.6)}{20} = 35.2 \div 20 = 1.76\)
   g. The MAD shows the data are fairly spread out.

2. a. least = 6; greatest = 45
   b. Intervals: 1–12, 13–24, 25–36, 37–48; the intervals are equal, and there are few enough that the histogram won’t be too big
   c. ![Histogram of spectator ages]
   d. Most of the spectators were under 24 years of age.

Skill: Finding Mean Absolute Deviation

1. 2.5
2. about 1.2
3. about 7.2
4. about 4.8
5. about 2.6
6. about 8.1

Skill: Reading Box Plots

7. 23, 24, 25, 28, 30; range = 7; IQR = 4
8. 1, 2, 7, 10, 16; range = 15; IQR = 8
9. 45, 48, 55, 58.5, 62; range = 17; IQR = 10.5
10. 78, 88, 96, 99, 106; range = 28; IQR = 11
Investigation 5 Check-Up
1. a. mean = 64.9 in.; median = 65.5 in.; no mode
   b. The IQR is 3 in., so most of the data are clustered around the median.
   d. Most of the data are clustered around the median, but the minimum value, 57 in., is much less than the rest of the data.
   e. 57: 7.9; 64: 0.9; 65: 0.1; 66: 1.1; 67: 2.1; 68: 3.1
   f. MAD = \((7.9 + 0.9 + 0.1 + 0.1 + 1.1 + 1.1 + 2.1 + 2.1 + 3.1) ÷ 10 = 19.4 ÷ 10 = 1.94\)
   g. Most height are very close to the median height.
   h. The mean increases from 64.9 to about 65.8; the median increases from 65.5 to 66; and there is still no mode.
2. a. b. The most common height is between 81 and 83 inches, and there are only 2 players shorter than 75 inches tall.